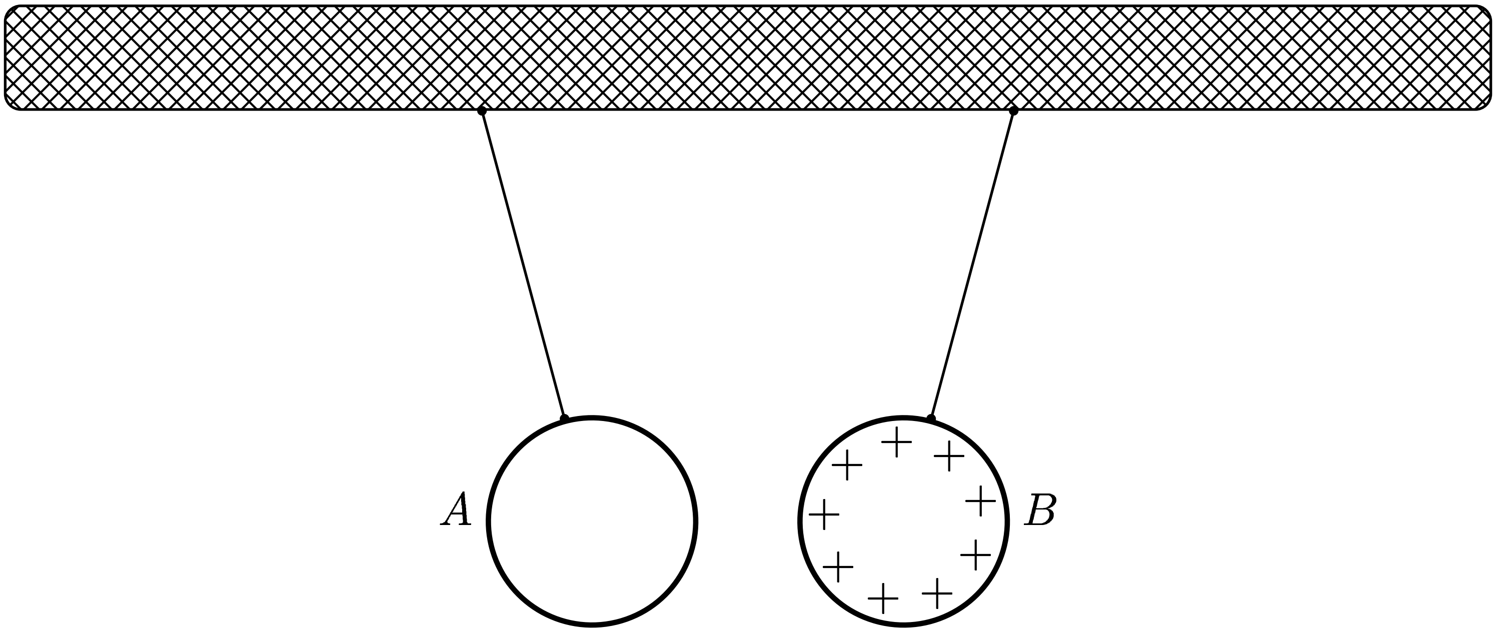
Given two conducting spheres A and B. There are positive charges on B; *i.e.*, $Q_B > 0$. The set up is in static equilibrium.



What is the sign of the net charge on A?

A) negative

B) positive

C) neutral

D) negative or neutral

E) positive or neutral

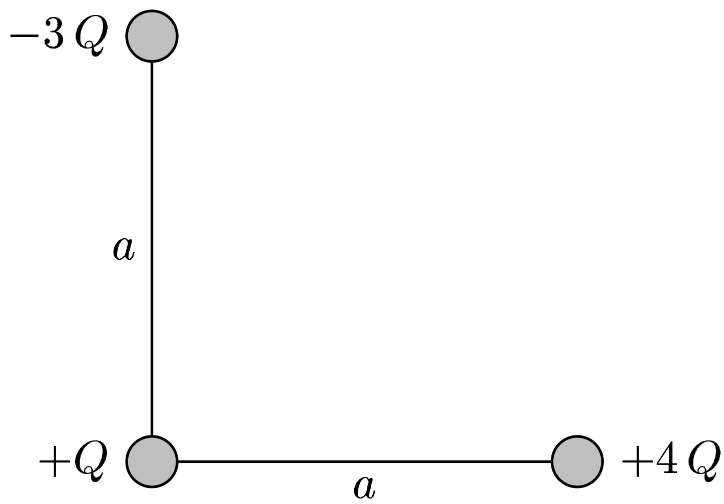
Coulomb's law is $\vec{F}_{AB} = k\,\frac{Q_A\,Q_B}{r^2}\,\hat{r}_{AB}$, which tells us that unlike charges attract. Consequently, if $Q_B > 0$ then $Q_A < 0$; *i.e.*, negative.

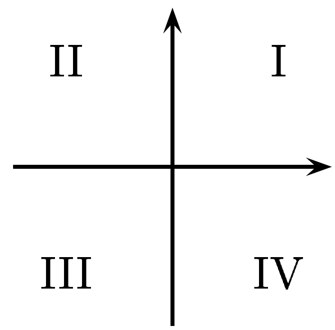
However if the net charge on sphere B is neutral, the influence of the positive charge on sphere A will polarize sphere B, such that the right-hand side of sphere A will become negative and the left-hand side of sphere A will become positive. This will produce an attractive force between sphere A and sphere B.

The answer is "negative or neutral".

Answer **D**.

Q is at the origin, $+4\,Q$ is on the positive x-axis a distance a from the origin, and $-3\,Q$ is on the positive y-axis a distance a from the origin.

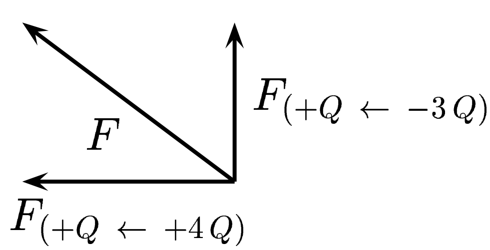




Determine which quadrant contains the electric force on the charge $+Q$ at the bottom left-hand corner (at the origin).

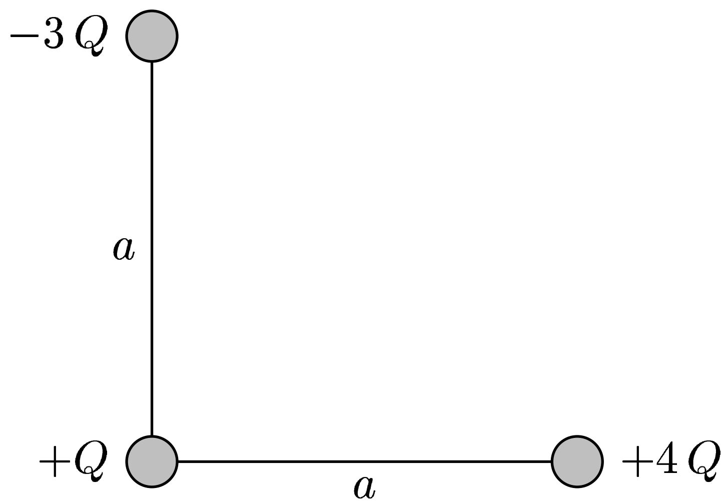
A) I
B) II
C) III
D) IV

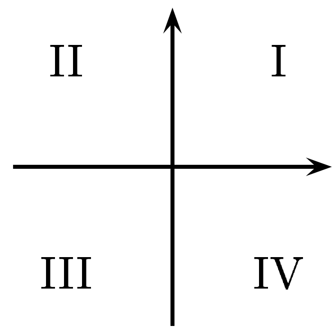
Coulomb's law is $\vec{F}_{AB} = k\,\frac{Q_A\,Q_B}{r^2}\,\hat{r}_{AB}$, which tells us that unlike charges attract and like charges repel.



Answer **B**.

Q is at the origin, $+4\,Q$ is on the positive x-axis a distance a from the origin, and $-3\,Q$ is on the positive y-axis a distance a from the origin.





Determine the magnitude of the electric force on the charge $+Q$ at the bottom left-hand corner (at the origin).

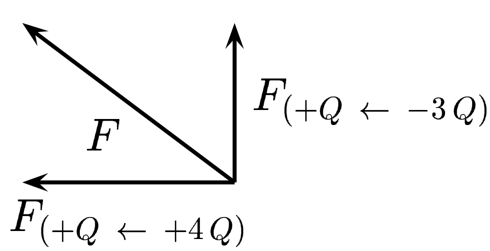
A) $\|\vec{F}\| = 3\,\frac{Q^2}{r^2}$.

B) $\|\vec{F}\| = 4\,\frac{Q^2}{r^2}$.

C) $\|\vec{F}\| = 5\,\frac{Q^2}{r^2}$.

D) $\|\vec{F}\| = \sqrt{5}\,\frac{Q^2}{r^2}$.

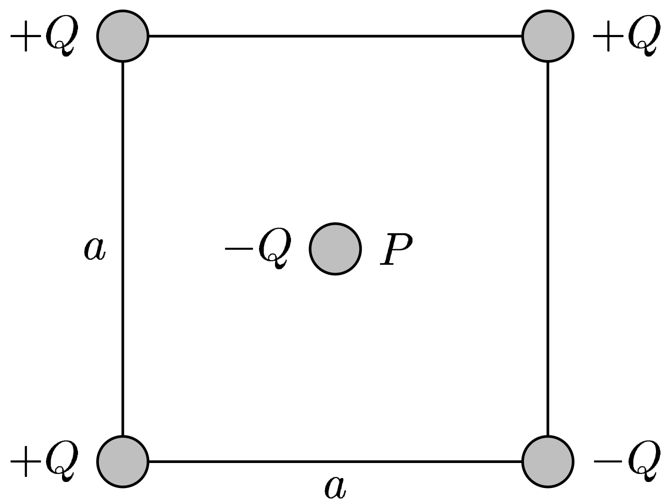
Coulomb's law is $\vec{F}_{AB} = k\,\frac{Q_A\,Q_B}{r^2}\,\hat{r}_{AB}$, which tells us that unlike charges attract and like charges repel.



$$\|\vec{F}\| = \sqrt{4^2+3^2}\,\frac{Q^2}{a^2} = 5\,\frac{Q^2}{r^2}.$$

Answer **C**.

Four point charges are located a distance a apart at the corners of a square.

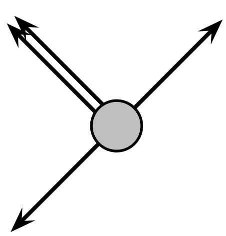


Determine the direction of the electric force on a negative charge $-Q$ located at the center of the square.

A) $\nearrow$ C) $\nwarrow$
B) $\searrow$ D) $\swarrow$

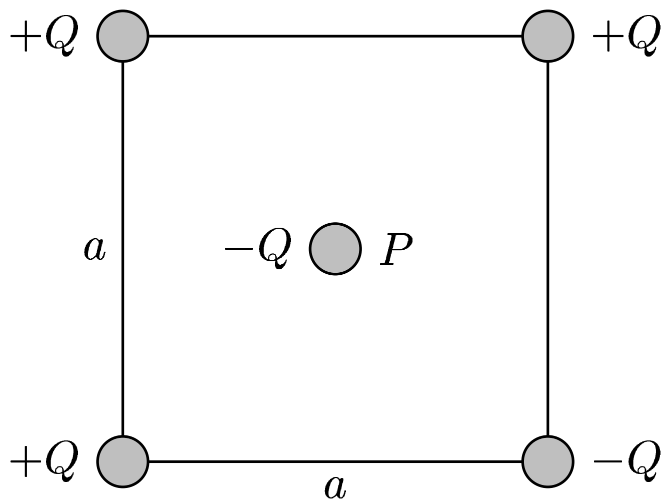
Coulomb's law is $\vec{F}_{AB} = k\,\dfrac{Q_A\,Q_B}{r^2}\,\hat{r}_{AB}$, which tells us that unlike charges attract and like charges repel.

$$\|\vec{F}\| = 2\,\frac{Q^2}{\left(\dfrac{a}{\sqrt{2}}\right)^2} = 4\,\frac{Q^2}{a^2}\,.$$



Answer **C**.

Four point charges are located a distance a apart at the corners of a square.



Determine the magnitude of the electric force on a negative charge $-Q$ located at the center of the square.

A) $\|\vec{F}\| = 4\,\dfrac{Q^2}{a^2}$.

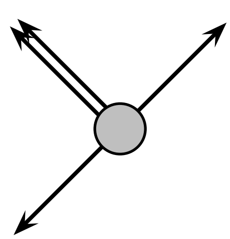
B) $\|\vec{F}\| = 2\,\dfrac{Q^2}{a^2}$.

C) $\|\vec{F}\| = \sqrt{2}\,\dfrac{Q^2}{a^2}$.

D) $\|\vec{F}\| = \dfrac{Q^2}{a^2}$.

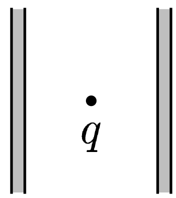
Coulomb's law is $\vec{F}_{AB} = k\,\dfrac{Q_A\,Q_B}{r^2}\,\hat{r}_{AB}$, which tells us that unlike charges attract and like charges repel.

$$\|\vec{F}\| = 2\,\frac{Q^2}{\left(\dfrac{a}{\sqrt{2}}\right)^2} = 4\,\frac{Q^2}{a^2}\,.$$



Answer **A**.

Imagine a charge in the middle between two parallel plate conductors. There is no net charge on the plates, and the plates are not connected to ground.



What will happen if the charge is moved a little away from the middle?

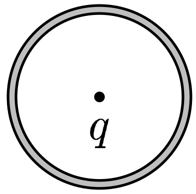
A) The charge will return to the middle.
B) The charge will remain stationary.
C) The charge will move away from the middle.
D) All of these can happen, depending on the size of the charge.

There will be an image-charge attracting it towards each metal surface. The charge will move towards the plate closest to it because the image-charge attracting it will be closest and thus stronger.

Any charge (free to move) will go toward the closest conductor it can find.

Answer **C**.

Imagine a charge in the center of a conducting, hollow sphere. There is no net charge on the sphere, and the sphere is not connected to ground.



What will happen if the charge is moved a little away from the center?

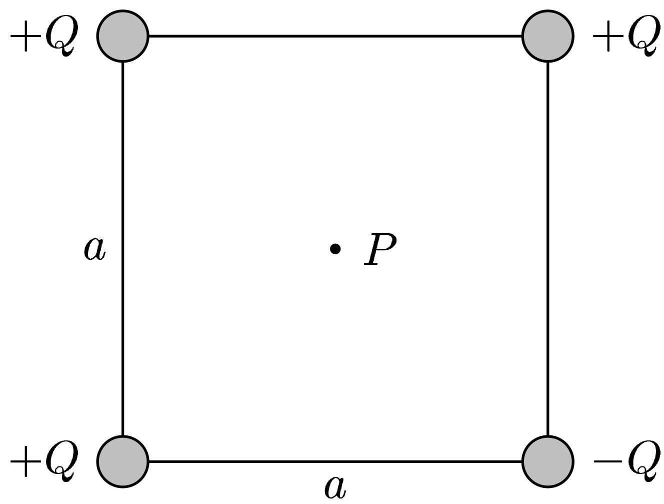
A) The charge will return to the center.
B) The charge will remain stationary.
C) The charge will move away from the center.
D) All of these can happen, depending on the size of the charge.

There will be an image-charge attracting it towards each metal surface. The charge will move towards the metallic surface closest to it because the image-charge attracting it will be closest and thus stronger.

Any charge (free to move) will go toward the closest conductor it can find.

Answer **C**.

Four point charges are located a distance a apart at the corners of a square.

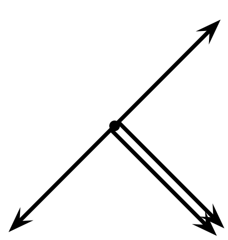


Determine the direction of the electric field at a point P located at the center of the square.

A) ↗ C) ↖
B) ↘ D) ↙

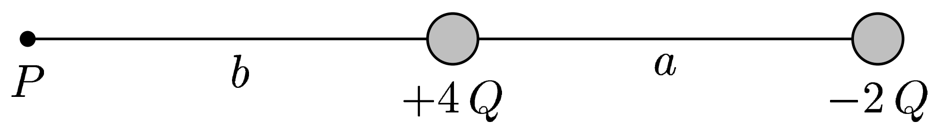
Coulomb's law is $\vec{E} = k\,\dfrac{Q}{r^2}\,\hat{r}$.

$$\|\vec{E}\| = 2\,\frac{Q}{\left(\dfrac{a}{\sqrt{2}}\right)^2} = 4\,\frac{Q}{a^2}\,.$$



Answer **B**.

Two point charges are located a distance a apart and lie on the x-axis. Point P is located a distance b from the charge $+4\,Q$ (the left-most charge).



At P, the direction of the electric field due to the $-2\,Q$ charge and the $+4\,Q$ charge are in opposite directions.

Compare the magnitude of the electric fields from the two charges at a point P to the left of the $+4\,Q$ charge on the x-axes.

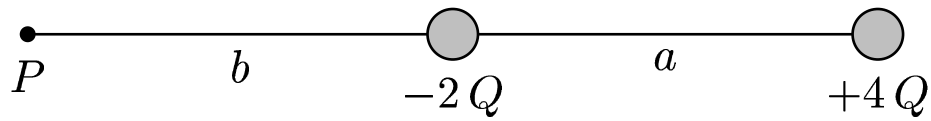
A) Only $E_{-2\,Q} > E_{+4\,Q}$ is possible.
B) Only $E_{-2\,Q} = E_{+4\,Q}$ is possible.
C) Only $E_{-2\,Q} < E_{+4\,Q}$ is possible.
D) All of the above are possible.
E) None of the above are possible.

Coulomb's law is $\vec{E} = k\,\frac{Q}{r^2}\,\hat{r}$.

$$\|\vec{E}_{+4\,Q}\| = k\,\frac{|+4\,Q|}{b^2}$$
$$\|\vec{E}_{-2\,Q}\| = k\,\frac{|-2\,Q|}{(a+b)^2} < k\,\frac{|-2\,Q|}{b^2} < k\,\frac{|+4\,Q|}{b^2}\,,$$

therefore only $E_{-2\,Q} < E_{+4\,Q}$ is possible.
Answer **C**.

Two point charges are located a distance a apart and lie on the x-axis. Point P is located a distance b from the charge $-2\,Q$ (the left-most charge).



At P, the direction of the electric field due to the $+4\,Q$ charge and the $-2\,Q$ charge are in opposite directions.

Compare the magnitude of the electric fields from the two charges at a point P to the left of the $-2\,Q$ charge on the x-axes.

A) Only $E_{+4\,Q} > E_{-2\,Q}$ is possible.
B) Only $E_{+4\,Q} = E_{-2\,Q}$ is possible.
C) Only $E_{+4\,Q} < E_{-2\,Q}$ is possible.
D) All of the above are possible.
E) None of the above are possible.

Coulomb's law is $\vec{E} = k\,\frac{Q}{r^2}\,\hat{r}$.

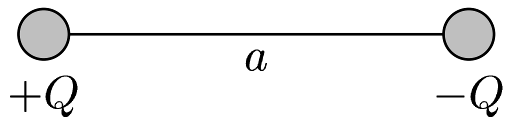
$$\begin{aligned} -k\,\frac{-2\,Q}{b^2} &= k\,\frac{+4\,Q}{(a+b)^2} \\ \frac{2}{b^2} &= \frac{4}{(a+b)^2} \\ \frac{a+b}{b} &= \sqrt{2} \\ b &= \frac{a}{\sqrt{2}-1}\,, \end{aligned}$$

therefore if P is closer to $-2\,Q$, then $\|\vec{E}_{-2\,Q}\| > \|\vec{E}_{+4\,Q}\|$ and if P is farther away from $-2\,Q$, then $\|\vec{E}_{-2\,Q}\| < \|\vec{E}_{+4\,Q}\|$.

Consequently, "All of the above are possible".

Answer **D**.

Two point charges are located a distance of a apart and lie on the x-axis.



Determine the electric field vector $\vec{E}$ due to $-Q$ at $+Q$.

A) $\|\vec{E}\| = -k\,\dfrac{Q}{a^2}$ direction : $\leftarrow$

B) $\|\vec{E}\| = +k\,\dfrac{Q}{a^2}$ direction : $\leftarrow$

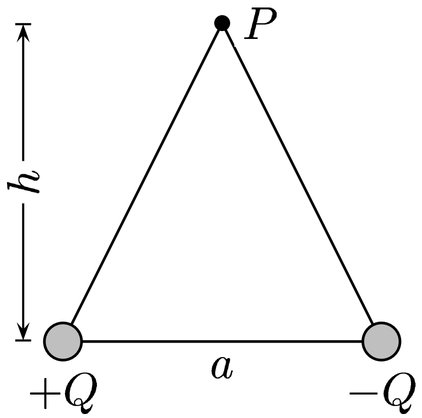
C) $\|\vec{E}\| = -k\,\dfrac{Q}{a^2}$ direction : $\rightarrow$

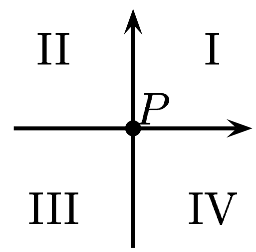
D) $\|\vec{E}\| = +k\,\dfrac{Q}{a^2}$ direction : $\rightarrow$

The magnitude of a vector $\|\vec{E}\|$ is always positive. Since the charges are of opposite sign $\|\vec{E}\| = +k\,\dfrac{Q}{a^2}$ direction : $\leftarrow$

Answer **B**.

Two point charges of equal magnitude are a distance of a apart and are located on the x-axes.





Determine the direction of the electric field at a point P located on a perpendicular bisector of a a distance h away from the x-axes.

A) The direction of $\vec{E}$ is in the I quadrant.

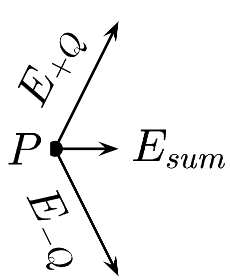
B) The direction of $\vec{E}$ is in the II quadrant.

C) The direction of $\vec{E}$ is in the III quadrant.

D) The direction of $\vec{E}$ is in the IV quadrant.

E) The direction of $\vec{E}$ is along the x-axes or y-axes.

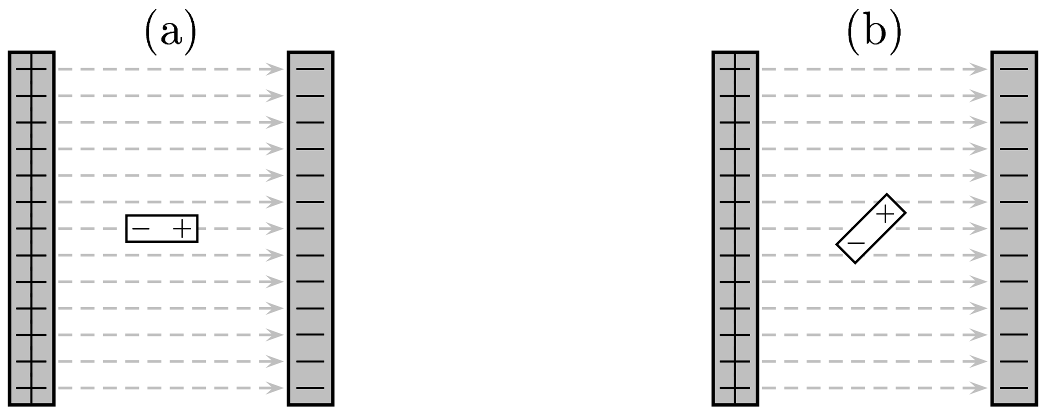
Coulomb's law is $\vec{E} = k\,\dfrac{Q}{r^2}\,\hat{r}$.



Therefore the direction of electric field $\vec{E}_{sum}$ at point P lies along the positive x-axes.

Answer **E**.

A dipole (electrically neutral) is placed in an external field.



For which situation(s) shown above is the net force on the dipole zero?

A) (a) only
B) (b) only
C) Both (a) and (b)
D) Neither (a) or (b)

Basic Concepts: Field patterns of point charge and parallel plates of infinite extent.

The force on a charge in the electric field is given by

$$\vec{F} = q\vec{E}$$

$$\Delta\vec{E} = \frac{k\Delta q}{r^2}\hat{r}$$

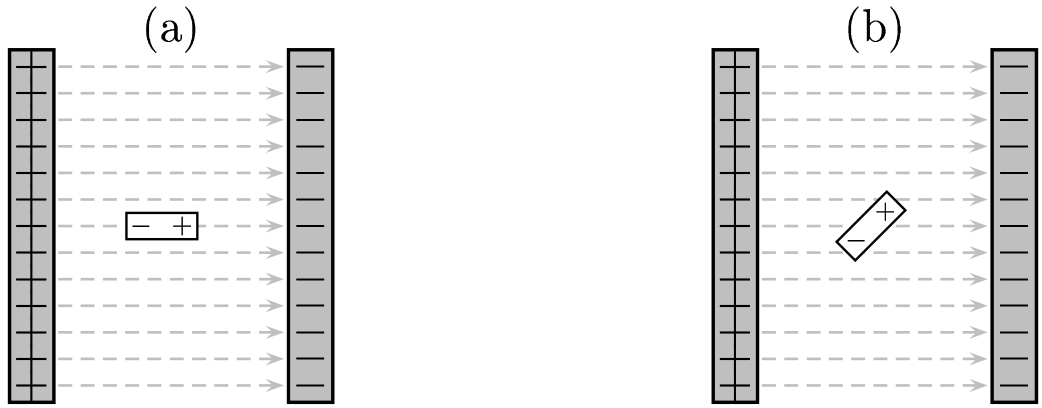
$$\vec{E} = \sum \Delta\vec{E}_i\,.$$

Symmetry of the configuration will cause some component of the electric field to be zero.

Solutions: The electric dipole consists of two equal and opposite charges separated by a distance. The electric fields are uniform for situations both Figs. (a) and (b). The force will be largest where the field is the strongest, but both ends of the dipole will have the same field strength. Consequently, there will be NO net force on (a) or (b).

Answer **C**.

A dipole (electrically neutral) is placed in an external field.



For which situation(s) shown above is the net torque on the dipole zero?

A) (a) only
B) (b) only
C) Both (a) and (b)
D) Neither (a) or (b)

Basic Concepts: Field patterns of point charge and parallel plates of infinite extent.

The force on a charge in the electric field is given by

$$\vec{F} = q\vec{E}$$

$$\Delta\vec{E} = \frac{k\Delta q}{r^2}\hat{r}$$

$$\vec{E} = \sum \Delta\vec{E}_i\,.$$

Symmetry of the configuration will cause some component of the electric field to be zero.

Solutions: The electric dipole consists of two equal and opposite charges separated by a distance. Only in Fig. (a), the electric field is along the direction of $\vec{r}$, where $\vec{r}$ is the vector between the pair of charges. Therefore the force $\vec{F}$ is also along $\vec{r}$. This will lead to zero torque, since

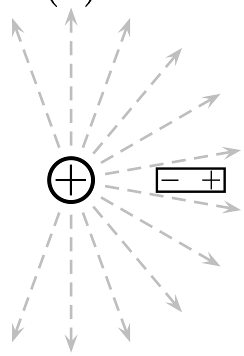
$$\vec{T} = \vec{r} \times \vec{F} \propto \vec{r} \times \vec{r} = 0\,.$$

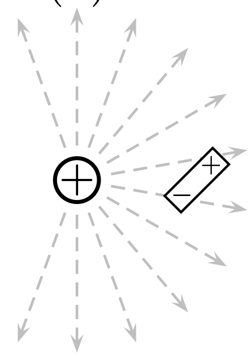
For Fig. (b), the torque on both charges are not equal, nonzero, and the net torque is nonzero.

Answer **A**.

A dipole (electrically neutral) is placed in an external field.

(a) (b)





For which situation(s) shown above is the net force on the dipole zero?

A) (a) only
B) (b) only
C) Both (a) and (b)
D) Neither (a) nor (b)

Basic Concepts: Field patterns of point charge and parallel plates of infinite extent.

The force on a charge in the electric field is given by

$$\vec{F} = q\vec{E}$$

$$\Delta\vec{E} = \frac{k\Delta q}{r^2}\hat{r}$$

$$\vec{E} = \sum \Delta\vec{E}_i \,.$$

Symmetry of the configuration will cause some component of the electric field to be zero.

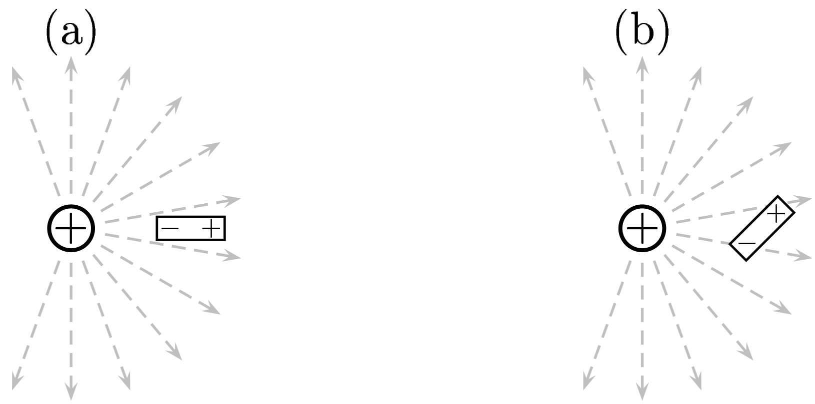
Gauss' law states:

$$\Phi_S = \oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

Solutions: The electric dipole consists of two equal and opposite charges separated by a distance. The electric fields are nonuniform for situations both Figs. (a) and (b). The force will be largest where the field is the strongest. Consequently, there will be a net force in both (a) and (b).

Answer **D**.

A dipole (electrically neutral) is placed in an external field.



For which situation(s) shown above is the net torque on the dipole zero?

A) (a) only
B) (b) only
C) Both (a) and (b)
D) Neither (a) nor (b)

Basic Concepts: Field patterns of point charge and parallel plates of infinite extent.

The force on a charge in the electric field is given by

$$\vec{F} = q\vec{E}$$

$$\Delta\vec{E} = \frac{k\Delta q}{r^2}\hat{r}$$

$$\vec{E} = \sum \Delta\vec{E}_i \,.$$

Symmetry of the configuration will cause some component of the electric field to be zero.

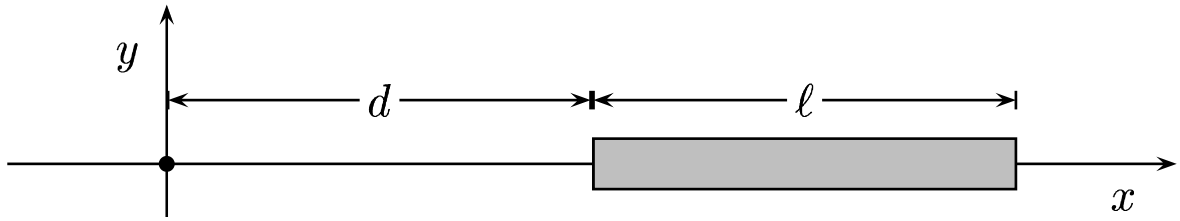
Solutions: The electric dipole consists of two equal strength poles a distance apart. Only in figure (a), the electric field is along the direction of $\vec{r}$, where $\vec{r}$ is the vector between the pair of charges. Therefore the force $\vec{F}$ is also along $\vec{r}$. This will lead to zero torque, since

$$\vec{T} = \vec{r} \times \vec{F} \propto \vec{r} \times \vec{r} = 0 \,.$$

For figures (b), the torque on both charges are nonzero and the resultant torques are also nonzero. opposite charges separated by a distance.

Answer **A**.

A rod with linear charge density $\lambda < 0$ and length ℓ lies along the x-axes with its left-hand end a distance d from the origin.



By inspection $\vec{E}$ is pointing along the positive x-axes, since the charge on the rod is negative $\lambda < 0$.

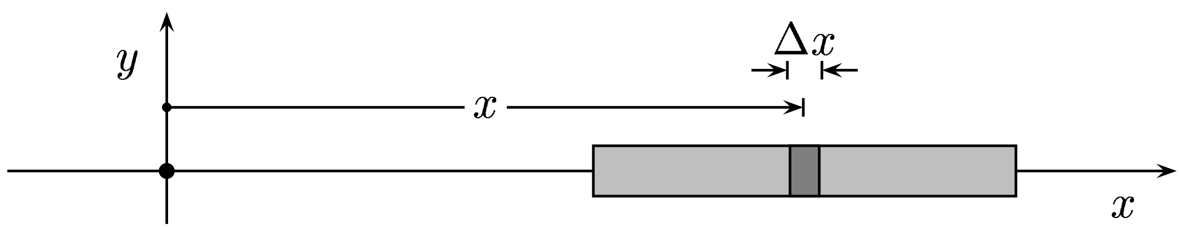
What is the magnitude of the electric field at the origin?

A) $\|\vec{E}\| = k\,\lambda \displaystyle\int_d^{d+\ell} \frac{1}{x^2}\,dx$

B) $\|\vec{E}\| = k\,\lambda \displaystyle\int_d^{\ell} \frac{1}{x^2}\,dx$

C) $\|\vec{E}\| = \dfrac{k}{\lambda} \displaystyle\int_d^{d+\ell} \frac{1}{x^2}\,dx$

D) $\|\vec{E}\| = \dfrac{k}{\lambda} \displaystyle\int_d^{\ell} \frac{1}{x^2}\,dx$



$$\text{Since} \quad \Delta E = k\,\frac{\Delta Q}{x^2} \quad \text{and} \quad \Delta Q = \lambda\,\Delta x$$

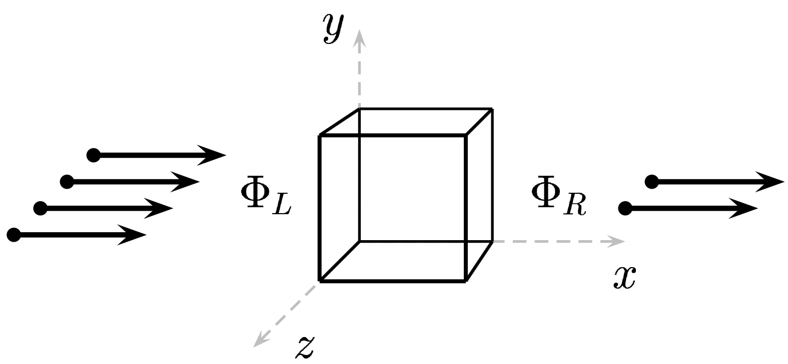
$$= k\,\frac{\lambda\,\Delta x}{x^2}, \quad \text{so}$$

$$E = \int \Delta E$$

$$= k\,\lambda \int_d^{d+\ell} \frac{1}{x^2}\,dx\,.$$

Answer **A**.

Consider the space of a cubic box.
The Electric field is parallel to the x-axis.
Flux entering from left: $|\Phi_L| = 4\,\mathrm{N\,m^2/C}$.
Flux leaving from right: $|\Phi_R| = 2\,\mathrm{N\,m^2/C}$.



Find Q_{encl}, the net charge enclosed.

A) $Q_{encl} = 4\,\epsilon_0$
B) $Q_{encl} = 2\,\epsilon_0$
C) $Q_{encl} = -2\,\epsilon_0$
D) $Q_{encl} = -4\,\epsilon_0$
E) $Q_{encl} = 0$

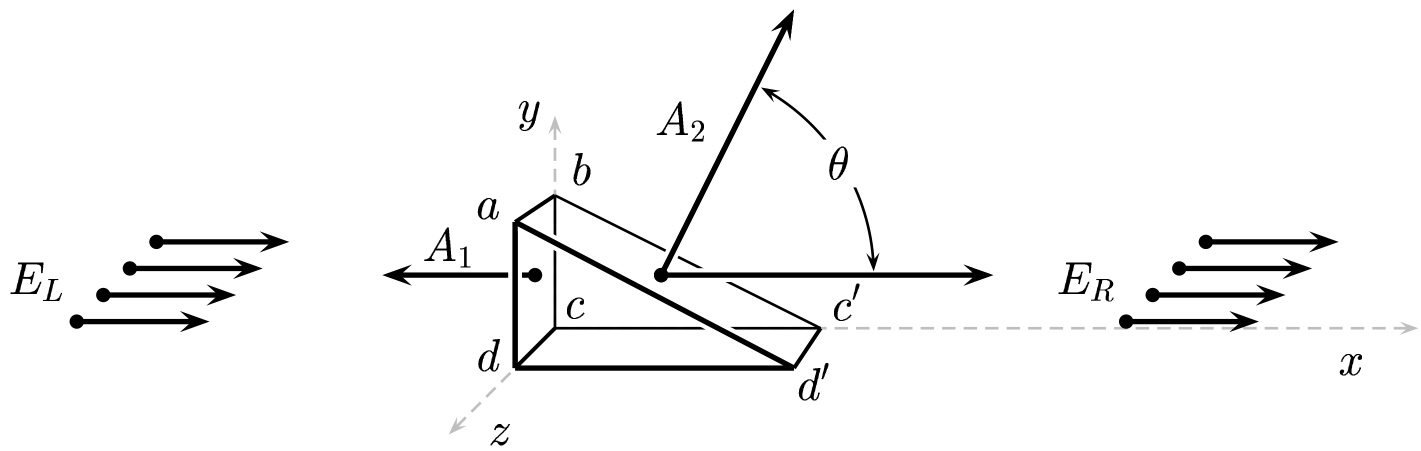
Gauss's Law states that $\Phi_S = \dfrac{Q_{encl}}{\epsilon_0}$.
Here Φ_S is the flux leaving the cubic region.

$$\begin{aligned}\frac{Q_{encl}}{\epsilon_0} &= \Phi_S \\ &= -|\Phi_L| + |\Phi_R| \\ &= -4 + 2 \\ &= -2\,, \quad \text{so} \\ Q_{encl} &= -2\,\epsilon_0\,.\end{aligned}$$

Answer **C**.

Given: A constant electric fields $\vec{E}$ along the x-direction.

The first rectangle $abcd$ has an area A_1 perpendicular to $\vec{E}$. The second rectangle $abc'd'$ has an area A_2 and it is inclined with an angle $\angle_{dad'} = \theta$.



Find Φ_2, the flux due to the field $\vec{E}$ through the second rectangle $abc'd'$.

A) $\Phi_2 = E\,A_2$

B) $\Phi_2 = E\,A_2 \cos\theta$

C) $\Phi_2 = E\,A_1 \cos\theta$

D) $\Phi_2 = E\,A_2 \sin\theta$

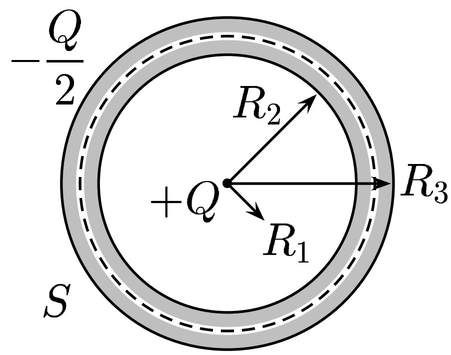
E) $\Phi_2 = E\,A_1 \sin\theta$

For the second rectangle, the projection of its area which is perpendicular to $\vec{E}$ is

$$A_\perp = \text{area}_{abc'd'} = A_2 \cos\theta\,.$$

Answer **B**.

Consider an electrostatic situation. A point charge Q is located at the center of a thick spherical conducting shell. The net charge on the shell is $-\frac{1}{2}Q$. Let S (dashed circular line) be a concentric spherical surface (Gaussian surface) with a radius r.



Find the flux Φ_S emanating through S, the Gaussian surface.

A) $\Phi_S = \frac{Q}{\epsilon_0}$

B) $\Phi_S = \frac{Q}{2\,\epsilon_0}$

C) $\Phi_S = \frac{3\,Q}{2\,\epsilon_0}$

D) $\Phi_S = 0$

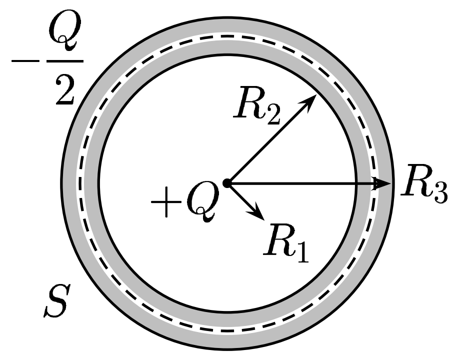
E) $\Phi_S = -\frac{Q}{\epsilon_0}$

For an electrostatic case, inside of a conductor or in a conducting medium, $e = 0$. This implies that $\Phi_S = \oint_S \vec{E} \cdot \vec{A} = 0$.

Answer **D**.

24.03-01˙Conducting˙Shell˙and˙Point˙Charge 2006-9-14

Consider an electrostatic situation. A point charge Q is located at the center of a thick spherical conducting shell. The net charge on the shell is $-\frac{1}{2}Q$. Let S (dashed circular line) be a concentric spherical surface (Gaussian surface) with a radius r.



What is the charge on the outer surface of the thick spherical conducting shell?

A) $Q_{outer\ surface} = -\frac{1}{2}\,Q$ B) $Q_{outer\ surface} = +\frac{1}{2}\,Q$

C) $Q_{outer\ surface} = -Q$ D) $Q_{outer\ surface} = +Q$

E) $Q_{outer\ surface} = -\frac{3}{2}\,Q$

For an electrostatic case, there must not be charge(s) inside of a conductor (otherwise $E_{inside} \neq 0$). So the charges can only reside on the inner surface and outer surface of the conducting shell. Since $\Phi_S = 0$, the enclosed charge $Q_{inner\ surface} + Q = 0$, thus $Q_{inner\ surface} = -Q$.

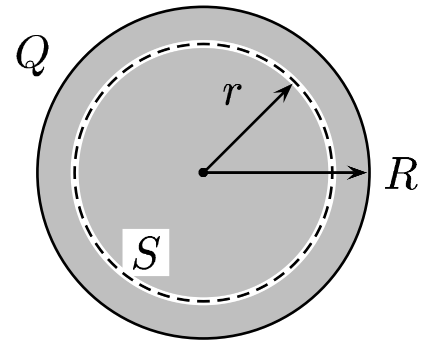
Since $Q^{net}_{shell} = Q_{inner\ surface} + Q_{outer\ surface}$, we have

$$\begin{aligned} Q_{outer\ surface} &= Q^{net}_{shell} - Q_{inner\ surface} \\ &= -\frac{1}{2}\,Q + Q \\ &= \frac{1}{2}\,Q\,. \end{aligned}$$

Answer **B**.

24.03-02˙Conducting˙Shell˙and˙Point˙Charge 2006-9-14

Consider an electrostatic situation. A sphere (insulator) has a uniform charge Q and radius R. Its charge density is therefore $\rho = \dfrac{Q}{V}$.



Construct a Gaussian surface S having concentric spherical surface with radius r.

Determine the charge enclosed by S.

A) $Q_{enclosed} = \dfrac{4}{3}\,\pi\, r^3\,\rho$

B) $Q_{enclosed} = \pi\, r^2\,\rho$

C) $Q_{enclosed} = 2\,\pi\, r^2\,\rho$

D) $Q_{enclosed} = \dfrac{4}{3}\,\pi\, R^3\,\rho$

E) $Q_{enclosed} = 2\,\pi\, R^2\,\rho$

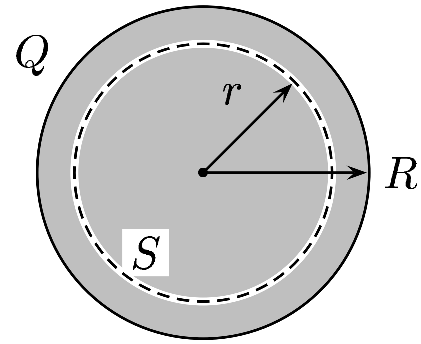
For an electrostatic case, the charge(s) inside of a conductor. The volume of a sphere is $V = \dfrac{4}{3}\,\pi\, r^3$.

$$\begin{aligned} Q_{inside} &= \rho\, V \\ &= \rho\,\frac{4}{3}\,\pi\, r^3 \\ &= \frac{4}{3}\,\pi\, r^3\, rho\,. \end{aligned}$$

Answer **A**.

24.03-03˙Uniformly˙Charged˙Sphere 2006-9-14

A sphere (insulator) has a uniform charge Q and radius R. Its charge density is therefore $\rho = \dfrac{Q}{V}$.



Construct a Gaussian surface S having concentric spherical surface with radius r.

Determine the magnitude of the electric field E at the Gaussian surface S.

A) $\|\vec{E}\| = \dfrac{4\,\rho}{3\,\epsilon_0}\, r^3$

B) $\|\vec{E}\| = \dfrac{2\,\rho}{3\,\epsilon_0}\, r$

C) $\|\vec{E}\| = \dfrac{1\,\rho}{2\,\epsilon_0}\, r^2$

D) $\|\vec{E}\| = \dfrac{1\,\rho}{3\,\epsilon_0}\, r$

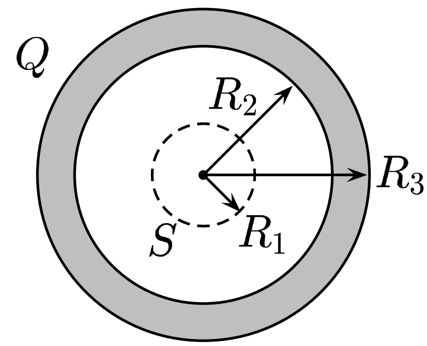
For an electrostatic case, the charge(s) inside of a conductor. The volume of a sphere is $V = \dfrac{4}{3}\,\pi\, r^3$.

$$
\begin{aligned}
Q_{inside} &= \rho\, V = \rho\, \frac{4}{3}\,\pi\, r^3\,, \quad \text{so} \\
E &= \frac{1}{4\,\pi\,\epsilon_0}\,\frac{Q_{inside}}{r^2} \\
&= \frac{1}{4\,\pi\,\epsilon_0}\,\frac{4\,\pi\, r^3\,\rho}{3\, r^2} \\
&= \frac{1}{3}\,\frac{\rho\, r}{\epsilon_0}\,.
\end{aligned}
$$

Answer **D**.

24.03-04˙Uniformly˙Charged˙Sphere 2004-3-24

A hollow thick spherical shell (made of an insulating material) has an inner radius of R_2 and an outer radius of R_3. The net charge on the shell is $Q > 0$, and the charge is uniformly distributed throughout the shell. Let S (dashed circular line) be a concentric spherical surface (Gaussian surface) with a radius R_1.



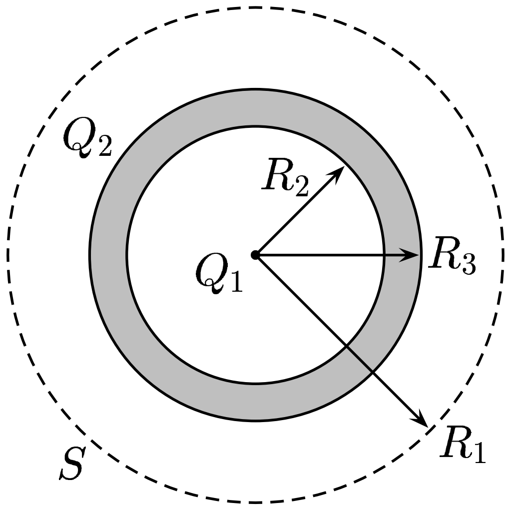
Find the direction of the electric field at a point R_1 from the center of the spherical conducting shell.

A) $\vec{E}$ is directed radially inward.
B) $\vec{E}$ is directed radially outward.
C) The direction of $\vec{E}$ is undetermined since $E = 0$.

Since the charge distribution is spherically symmetric, $\|\vec{E}\|$ must be the same everywhere on S. And by symmetry $\vec{E}$ must be directed radially, either outward or inward. However there is no charge enclosed in the Gaussian surface, therefore $\Phi_S = \oint_S \vec{E} \cdot \vec{A} = 0$, or specifically $E = 0$.

Answer **C**.

Consider an electrostatic situation. A point charge $Q_1 > 0$ is located at the center of a hollow thick spherical shell (made of an insulating material) that has an inner radius of R_2 and an outer radius of R_3. Naturally, the charge on the shell's inner surface is $-Q_1$, and the charge on the shell's outer surface is $Q_2 > 0$. Let S (dashed circular line) be a concentric spherical surface (Gaussian surface) with a radius R_1.



Find E_1, the magnitude of the radial electric field vector at the surface of the Gaussian surface S, which is a distance R_1 from the center of the spherical conducting shell.

A) $\vec{E} = \dfrac{1}{4\,\pi\,\epsilon_0}\,\dfrac{Q_1}{R_1^2}$

B) $\vec{E} = \dfrac{1}{4\,\pi\,\epsilon_0}\,\dfrac{Q_2}{R_1^2}$

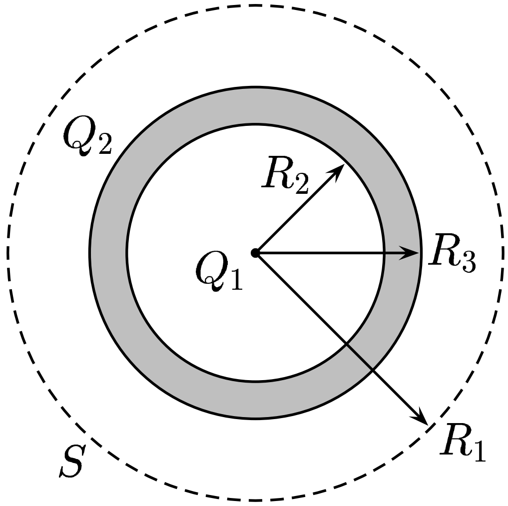
C) $\vec{E} = \dfrac{1}{4\,\pi\,\epsilon_0}\,\dfrac{Q_1 + Q_2}{R_1^2}$

Since the charge distribution is spherically symmetric, $\|\vec{E}\|$ must be the same everywhere on S. And by symmetry $\vec{E}$ must be directed radially, either outward or inward. However there is a charge enclosed in the Gaussian surface, therefore $\Phi_S = \oint_S \vec{E} \cdot \vec{A} = \dfrac{Q_1 - Q_1 + Q_2}{\epsilon_0}$, or specifically $\vec{E} = \dfrac{1}{4\,\pi\,\epsilon_0}\,\dfrac{Q_2}{R_1^2}$.

Answer **B**.

24.03-06˙Charged˙Spherical˙Shell 2006-9-14

A point charge $Q_1 > 0$ is located at the center of a hollow thick spherical shell (made of an insulating material) that has an inner radius of R_2 and an outer radius of R_3. The net charge on the shell is Q_2 and the charge is uniformly distributed throughout the shell. Let S (dashed circular line) be a concentric spherical surface (Gaussian surface) with a radius R_1.



Find E_1, the magnitude of the radial electric field vector at the surface of the Gaussian surface S, which is a distance R_1 from the center of the spherical conducting shell.

A) $\vec{E} = \dfrac{1}{4\,\pi\,\epsilon_0}\,\dfrac{Q_1}{R_1^2}$

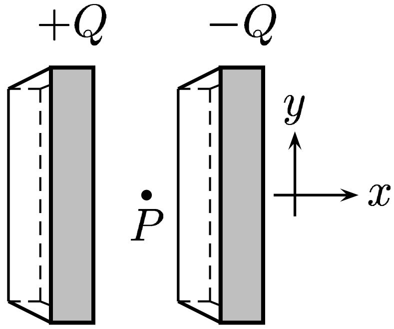
B) $\vec{E} = \dfrac{1}{4\,\pi\,\epsilon_0}\,\dfrac{Q_2}{R_1^2}$

C) $\vec{E} = \dfrac{1}{4\,\pi\,\epsilon_0}\,\dfrac{Q_1 + Q_2}{R_1^2}$

Since the charge distribution is spherically symmetric, $\|\vec{E}\|$ must be the same everywhere on S. And by symmetry $\vec{E}$ must be directed radially, either outward or inward. However there is a charge enclosed in the Gaussian surface, therefore $\Phi_S = \oint_S \vec{E} \cdot \vec{A} = \dfrac{Q_1 + Q_2}{\epsilon_0}$, or specifically $\vec{E} = \dfrac{1}{4\,\pi\,\epsilon_0}\,\dfrac{Q_1 + Q_2}{R_1^2}$.

Answer **C**.

Consider an electrostatic situation. A parallel plate system has a plate charge $+Q$ on the left-hand plate and a plate charge $-Q$ on the right-hand plate. Each plate has an area A.



Determine the the electric field E_{gap} at P, within the gap.

A) $\vec{E} = \dfrac{Q}{\epsilon_0 A}$, to the right.

B) $\vec{E} = \dfrac{Q}{\epsilon_0 A}$, to the left.

C) $\vec{E} = \dfrac{2Q}{\epsilon_0 A}$, to the right.

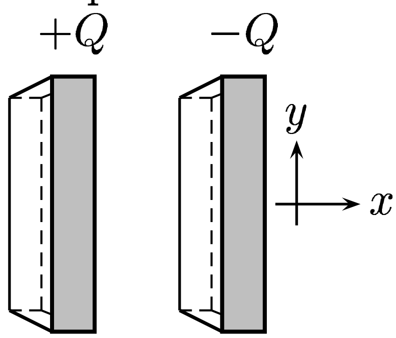
D) $\vec{E} = \dfrac{2Q}{\epsilon_0 A}$, to the left.

The areal charge density is $\sigma = \dfrac{Q}{A}$, therefore

$$E_{gap} = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}.$$

Answer **A**.

Consider an electrostatic situation. A parallel plate system has a plate charge $+Q$ on the left-hand plate and a plate charge $-Q$ on the right-hand plate. Each plate has an area A.



Determine the the force F the right-hand plate exerts on the left-hand plate.

A) $\vec{F}_{left} = \dfrac{Q^2}{\epsilon_0\, A}$, to the right.

B) $\vec{F}_{left} = \dfrac{Q^2}{\epsilon_0\, A}$, to the left.

C) $\vec{F}_{left} = \dfrac{Q^2}{2\,\epsilon_0\, A}$, to the right.

D) $\vec{F}_{left} = \dfrac{Q^2}{2\,\epsilon_0\, A}$, to the left.

The areal charge density is $\sigma = \dfrac{Q}{A}$, therefore

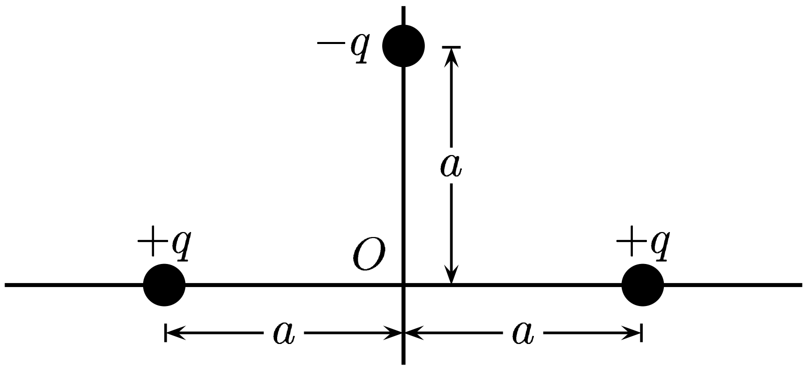
$$E_{gap} = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0\, A}\,.$$

The electric field due to be right-hand plate alone contributes to one-half of the total field in the gap; *i.e.*,

$$E_{left} = \frac{\sigma}{2\,\epsilon_0} = \frac{Q}{2\,\epsilon_0\, A} \quad \Rightarrow \quad \vec{F}_{left} = \frac{Q^2}{2\,\epsilon_0\, A}\,, \quad \text{to the right}\,.$$

Answer **C**.

Three point charges are placed at equal distance from O.



Find the potential at the origin O.

A) $V = -k\,\frac{q}{a}$

B) $V = +3\,k\,\frac{q}{a}$

C) $V = +k\,\frac{q}{a}$

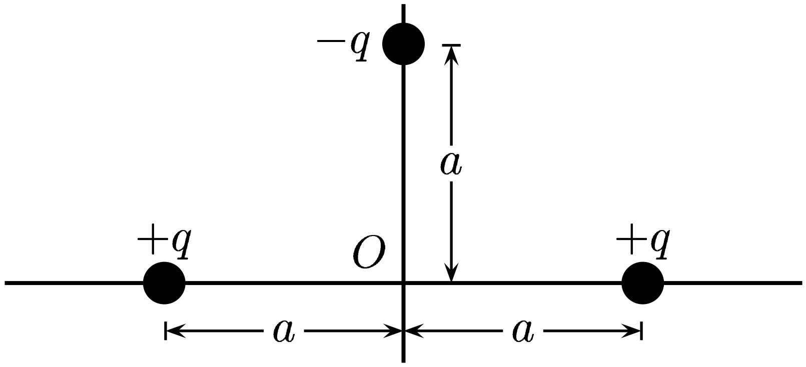
D) $V = -3\,k\,\frac{q}{a}$

At the origin O, we have

$V = V_0 = \frac{k\,q}{a} + \frac{k\,q}{a} - \frac{k\,q}{a} = \frac{k\,q}{a}$.

Answer **C**.

Three point charges are placed at equal distance from O.



Find the potential energy required to bring these charge from infinity to the positions shown above.

A) $U = k\,q^2\left[\frac{1}{\sqrt{2}} - 1\right]$

B) $U = k\,q^2\left[\sqrt{2} - \frac{1}{2}\right]$

C) $U = k\,q^2\left[\frac{1}{2} - \sqrt{2}\right]$

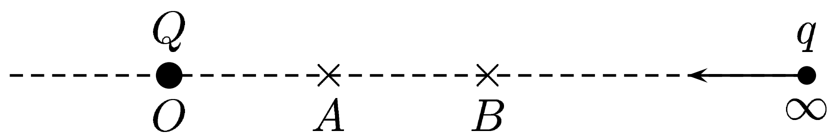
D) $U = k\,q^2\left[1 - \frac{1}{\sqrt{2}}\right]$

$$\begin{aligned} U &= -k\left[\frac{(-q)(+q)}{\sqrt{2}\,a} + \frac{(-q)(+q)}{\sqrt{2}\,a} + \frac{(+q)(+q)}{2\,a}\right] \\ &= -\frac{k\,q^2}{a}\left[-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} + \frac{1}{2}\right] \\ &= k\,q^2\left[\frac{1}{2} - \sqrt{2}\right] . \end{aligned}$$

Answer **C**.

A positive point charge Q is located at O.

Given: The work in bringing another positive charge q from infinity to the point A is $W_{\infty \to A} = 1$ J.



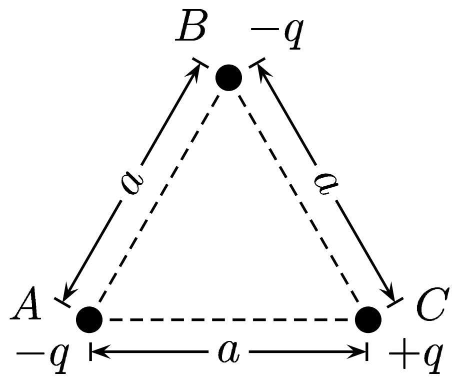
Find the work required in bringing the same q from infinity to the point B, where $\overline{OB} = 2\,a$, with $a = \overline{OA}$.

A) $W = \frac{1}{4}$ J
B) $W = \frac{1}{2}$ J
C) $W = 1$ J
D) $W = 2$ J
E) $W = 4$ J

$W_{\infty \Rightarrow B} = U|_B = k\,\frac{Q\,q}{2a} = \frac{1}{2}\,k\,\frac{Q\,q}{a}$. Here $k\,\frac{Q\,q}{a} = 1$ J, since it is the potential energy at A. So $W_{\infty \Rightarrow B} = \frac{1}{2}$ J$= 0.5$ J.

Answer **B**.

Three charges are located at the vertexes of an equilateral triangle, see sketch.



Excluding the charge at A, determine the direction of electric field vector and the potential at A.

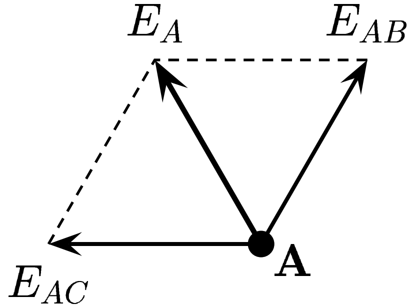
A) The direction of $\vec{E}_A$ is ↘ and the potential is $V_A = 0\,.$

B) The direction of $\vec{E}_A$ is ↖ and the potential is $V_A = -\dfrac{2\,k\,q}{a}\,.$

C) The direction of $\vec{E}_A$ is ↖ and the potential is $V_A = 0\,.$

D) The direction of $\vec{E}_A$ is ↘ and the potential is $V_A = -\dfrac{2\,k\,q}{a}\,.$

At A, the vector diagram of $E_{AB} + E_{AC}$ is given by

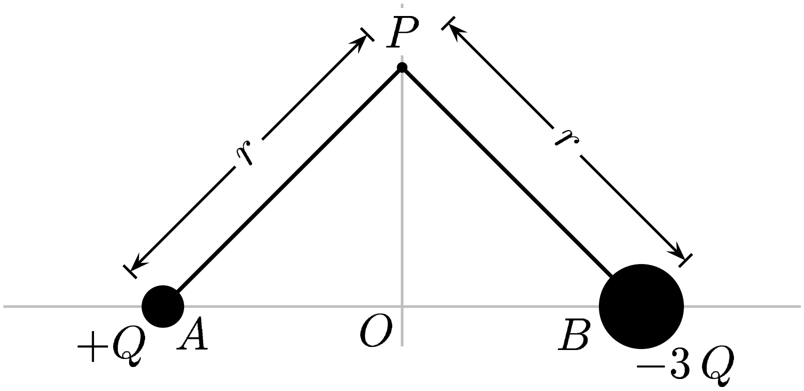


$$V_A = V_{AB} + V_{AC} = -\frac{k\,q}{a} + \frac{k\,q}{a} = 0.$$

Answer **C**.

25.03-04 Three Charges at Corners of an Equilateral Triangle 2004-3-24

Two charges are located on the x-axis.



Find the potential V_P, at the point P on the y-axis.

A) $V_P = +\dfrac{2\,k\,Q}{r}$

B) $V_P = -\dfrac{2\,k\,Q}{r}$

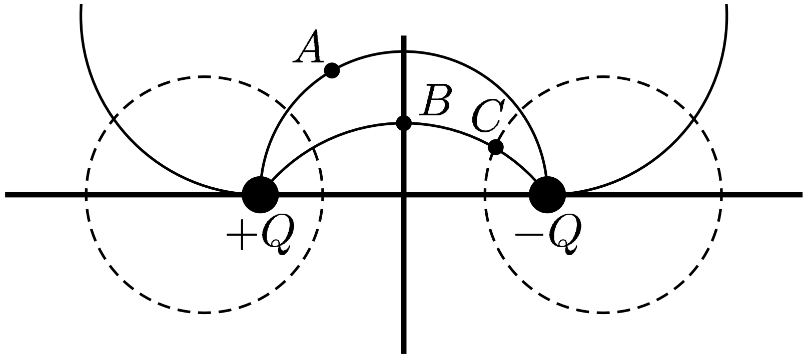
C) $V_P = -\dfrac{4\,k\,Q}{r}$

D) $V_P = +\dfrac{4\,k\,Q}{r}$

$$V_P = \frac{k\,Q_1}{r} + \frac{k\,Q_2}{r} = \frac{k\,Q}{r} - \frac{3\,k\,Q}{r} = -\frac{2\,k\,Q}{r}\,.$$

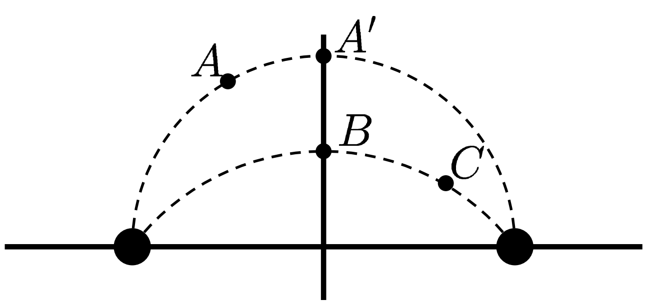
Answer **B**.

The field pattern and a pair of equipotential curves of a dipole are shown in the sketch.



Compare potentials at A, B and C.

A) $V_A > V_B$ and $V_B > V_C$.
B) $V_A > V_B$ and $V_B = V_C$.
C) $V_A < V_B$ and $V_B < V_C$.
D) $V_A < V_B$ and $V_B = V_C$.

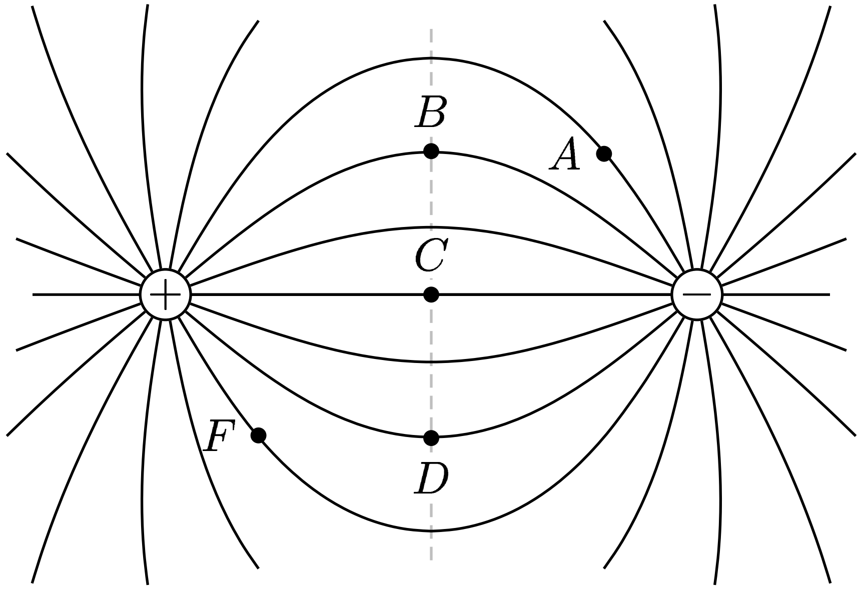


By inspection on the sketch above,

$$V_A > V_{A'}\,,$$
$$V_{A'} = V_B = 0\,, \quad \text{and}$$
$$V_B > V_C\,.$$

Answer **A**.

A dipole field pattern is shown in the figure. Consider various relationships between the electric potential V at different points given in the figure.



Which one of the following expressions is correct?

A) $V_B = V_D > V_C$
B) $V_B > V_C < V_D$
C) $V_B = V_D < V_C$
D) $V_A < V_C < V_F$
E) $V_A > V_C > V_F$

For a dipole system, the total potential at any place is the sum of potentials due to one positive point charge and one negative point charge (Superposition Principle).

From symmetry considerations, it is easy to see that the electric field lines are perpendicular to a line which passes through the midpoint C and points B and D.

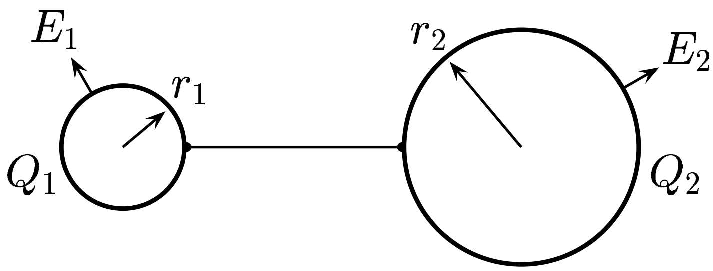
No work needs to be done to move a positive test charge along the mid-plane because the force and the displacement are perpendicular to each other. Therefore, $V_B = V_C = V_D$

Furthermore, moving along the direction of a electric field line (*i.e.*, moving in the direction from positive charge to negative charge along the electric field line) always lowers the electric potential, because the electric field will do positive work to a positive test charge in order to lower its electric potential energy.

Therefore, $V_A < V_B$ by considering the line going from B to A, and $V_D < V_F$ by considering the line going from F to D. Therefore, $V_A < V_C < V_F$

Answer **D**.

Two conducting spheres are far apart and are connected by a wire.
Assume: $V_1 \approx \dfrac{k\,Q_1}{r_1}$, $V_2 \approx \dfrac{k\,Q_2}{r_2}$.

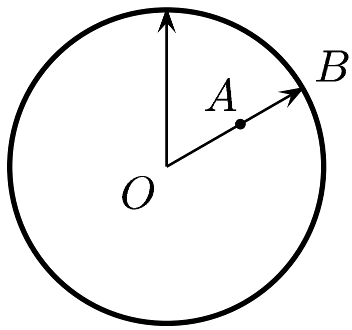


Compare the charges on the two spheres; *i.e.*, Q_1 vs Q_2.

A) $Q_1 > Q_2$
B) $Q_1 = Q_2$
C) $Q_1 < Q_2$

$V_1 = \dfrac{k\,Q_1}{r_1} = V_2 = \dfrac{k\,Q_2}{r_2}$. So $\dfrac{Q_1}{Q_2} = \dfrac{r_1}{r_2} < 1$, or $Q_1 < Q_2$.
Answer **C**.

Given a uniformly charged sphere with a total charge Q and a radius R. It can be shown that the electric field $E = \frac{\rho\, r}{3\, \epsilon_0}$.



Find the potential difference ΔV between A, where $OA = r < R$ and B, the point along the same radial line on the surface of the sphere.

A) $\Delta V = -\int_R^r E\, dr$

B) $\Delta V = \int_R^r E\, dr$

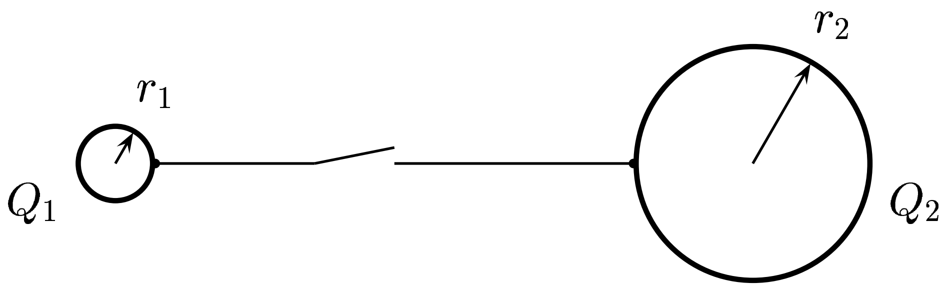
C) $\Delta V = E\,(R - r)$

$\Delta V = -E \cdot \Delta s$. For the present case E depends on r. So the potential difference must be evaluated through an integral. By inspection, answer **A** gives the desired positive potential difference.

Answer **A**.

Given: Two conducting spheres separated by a large distance are connected by a wire with an open switch. One is smaller than the other; *i.e.*, $r_1 < r_2$. Each has a positive charge $Q_1 = Q_2 = Q$ on it. Being far apart, the potentials on the spheres are assumed to be approximately given by

$$V_1 \approx k\,\frac{Q_1}{r_1}, \quad \text{and} \quad V_2 \approx k\,\frac{Q_2}{r_2}$$



When the switch is closed, describe the direction of the "apparent flow" of positive charges.

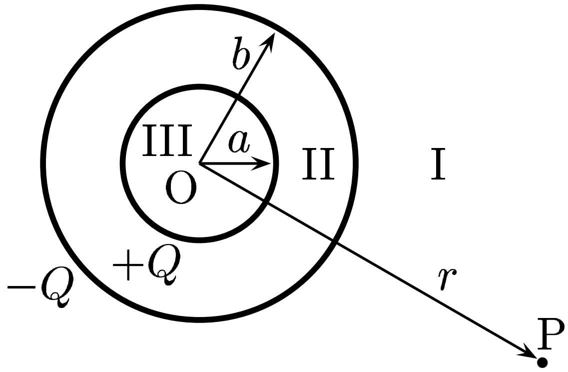
Note: Technically speaking, electrons are the ones which are flowing in the opposite direction.

A) Positive charges flow from sphere #1 to sphere #2.
B) Positive charges flow from sphere #2 to sphere #1.
C) No flow due to same charges on both bodies.

Since $r_1 < r_2$, from the equation above $V_1 > V_2$. The flow of positive charges is from sphere #1 to sphere #2. We mention once again, what really happens is that the negative charges flow from sphere #2 to sphere #1.

Answer **A**.

Given: Two thin concentric conducting spherical shells with charges Q and $-Q$ on the inner and outer shells respectively.



Find V at P in region I, which is a distance $\mathbf{r}$ from O.

A) $V = k\,\dfrac{Q}{r}$

B) $V = 0$

C) $V = -k\,\dfrac{Q}{r}$

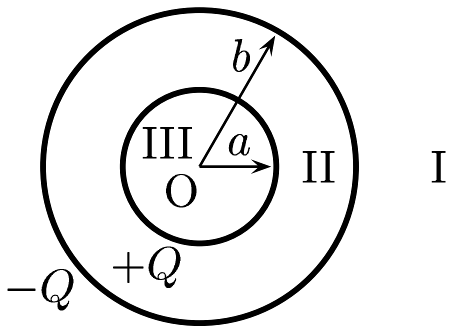
D) none of the above

In region I, using the superposition principle,

$$V = V_{inner} + V_{outer} = k\,\frac{Q}{r} - k\,\frac{Q}{r} = 0\,.$$

Answer **B**.

Given: Two thin concentric conducting spherical shells with charge Q on the inner shell (with radius a) and charge $-Q$ on the outer shell (with radius b).



Find $\Delta V_{ab} = V_b - V_a$.

A) $\Delta V = k\,\dfrac{Q}{b-a}$

B) $\Delta V = k\,\dfrac{Q}{b} - k\,\dfrac{Q}{a}$

C) $\Delta V = k\,\dfrac{Q}{a} + k\,\dfrac{Q}{b}$

D) $\Delta V = k\,\dfrac{Q}{a} - k\,\dfrac{Q}{b}$

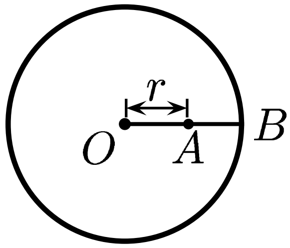
E) $\Delta V = 0$

In region II,

$$\Delta V_{ab} = V_b - V_a = -\int_\infty^b \vec{E}\cdot\vec{s}\,ds + \int_\infty^a \vec{E}\cdot\vec{s}\,ds = -\int_a^b \vec{E}\cdot\vec{s}\,ds$$
$$= -\frac{k\,Q}{r}\bigg|_a^b = k\,\frac{Q}{a} - k\,\frac{Q}{b}\,.$$

Answer **D**.

Consider a conducting sphere with a radius R, and charge Q.
It is in electrostatic equilibrium.



Find the potential V_A at A, $\overline{OA} = r < R$, and the potential V_O at O.

A) $V_A = k\,\dfrac{Q}{r}$ and $V_O = \infty$.

B) $V_A = 0$ and $V_O = 0$.

C) $V_A = k\,\dfrac{Q}{R}$ and $V_O = k\,\dfrac{Q}{R}$.

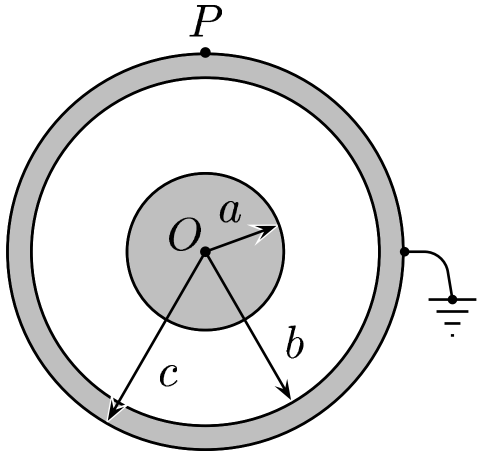
D) $V_A = k\,\dfrac{Q}{R}$ and $V_O = \infty$.

Being inside of an equipotential body, $V_O = V_A = V_B = \dfrac{k\,Q}{R}$.

Answer **C**.

Given: A spherical capacitor, see sketch.

It consists of an inner conducting sphere with a radius "a", and a concentric conducting shell with an inner radius "b" and an outer radius "c". The shell is grounded. There is a positive charge $+Q$ on the inner sphere.



Determine the magnitude of the field at the point P located at the top on the outer surface of the shell.

A) $E_A = k\,\dfrac{Q}{b^2}$

B) $E_A = k\,\dfrac{Q}{c^2}$

C) $E_A = 0$

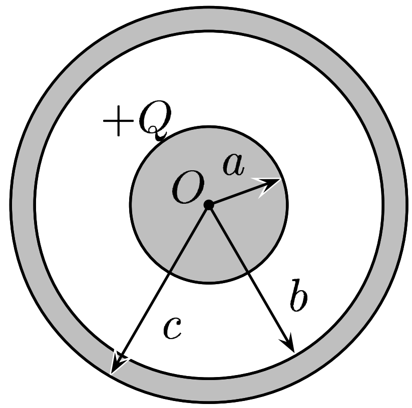
Suppose there were charges at the surface of the shell, there would be an electric field perpendicular to the surface, since

$$E_\perp = \frac{\sigma_{surface}}{\epsilon_0}\,.$$

In turn, there would be a charge flow between the surface of the shell and the ground. This is contrary to the fact that the shell is grounded; *i.e.*, there is no potential difference between the shell and the ground. So there can be no field at the surface.

Answer **C**.

An ungrounded spherical capacitor has a sphere and a concentric shell. Both are conductors. The charge on the sphere is $+Q$. The net charge on the shell is zero.



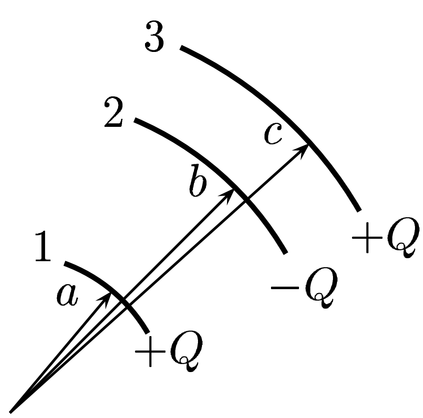
Find the potential V_0 at the origin.

A) $V_O = 0$

B) $V_O = k\,\dfrac{Q}{a}$

C) $V_O = k\,Q\left(\dfrac{1}{a} - \dfrac{1}{b} + \dfrac{1}{c}\right)$

There are 3 concentric spherical charge distributions:



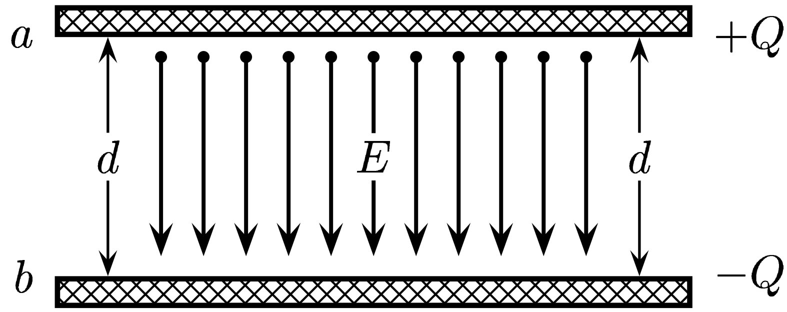
The superposition principle implies that at O

$$V_O = V_a + V_b + V_c = k\,Q\left(\frac{1}{a} - \frac{1}{b} + \frac{1}{c}\right).$$

Answer **C**.

26.02-03˙Potential˙at˙O˙of˙an˙Ungrounded˙System 2004-3-24

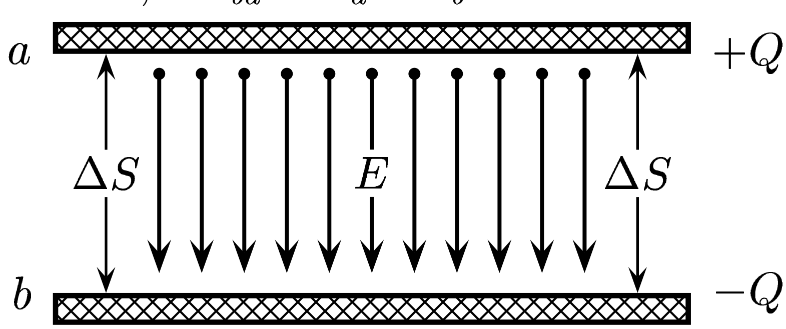
Two plates a and b are separated by a distance d. The "plate charge" is Q. Between the gap E is constant.



Find the potential difference $\Delta V_{ba} = V_a - V_b$.

A) $\Delta V_{ba} = E\,d$
B) $\Delta V_{ba} = -E\,d$
C) $\Delta V_{ba} = \dfrac{E\,d}{2}$
D) $\Delta V_{ba} = -\dfrac{E\,d}{2}$

From the sketch, $\Delta V_{ba} = V_a - V_b = -E\,\Delta S\,\cos 180^\circ = E\,d$.

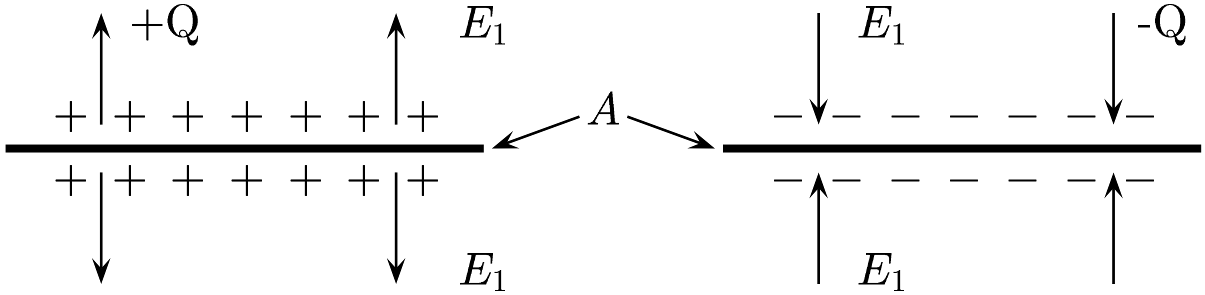


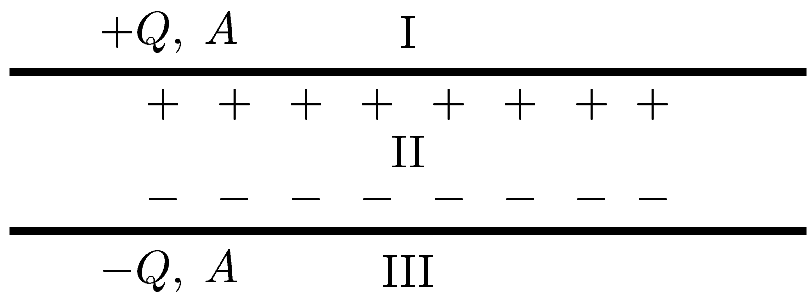
An independent check, we recall that the natural tendency for a positive charge is to move along E. So $V_a > V_b$. The sign is therefore correct.

Answer **A**.

26.02-04˙A˙Parallel˙Plate˙Capacitor 2004-3-24

Given 1-plate pattern, $E_1 = \dfrac{Q}{2\,\epsilon_0\,A}$:





Find electric fields E of parallel plate system in I, II and III.

A) $E_{\rm I} = 0$ and $E_{\rm II} = 2\,E_1 \downarrow$ and $E_{\rm III} = 0$.

B) $E_{\rm I} = E_1 \uparrow$ and $E_{\rm II} = 2\,E_1 \uparrow$ and $E_{\rm III} = E_1 \downarrow$.

C) $E_{\rm I} = E_1 \uparrow$ and $E_{\rm II} = E_1 \uparrow$ and $E_{\rm III} = E_1 \downarrow$.

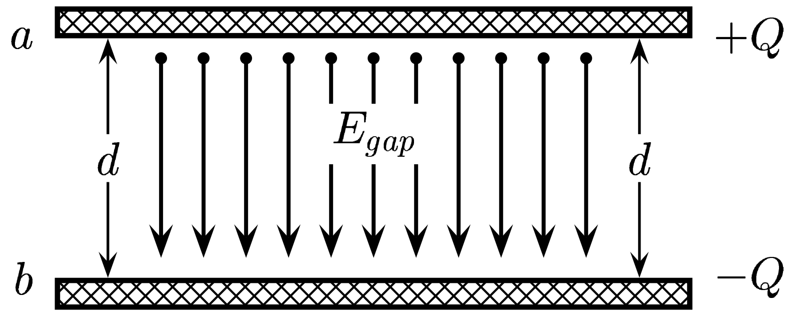
Apply the superposition principle

	Top plate	Bottom plate	Both plates
$E_{\rm I}$	$+E_1$	$-E_1$	0
$E_{\rm II}$	$-E_1$	$-E_1$	$-2\,E_1$
$E_{\rm III}$	$-E_1$	$+E_1$	0

Answer **A**.

A parallel plate system has a plate charge Q.

Within the gap $E_{\text{gap}} = \dfrac{\sigma_{\text{plate}}}{\epsilon_0} = \dfrac{Q}{\epsilon_0\, A}$.

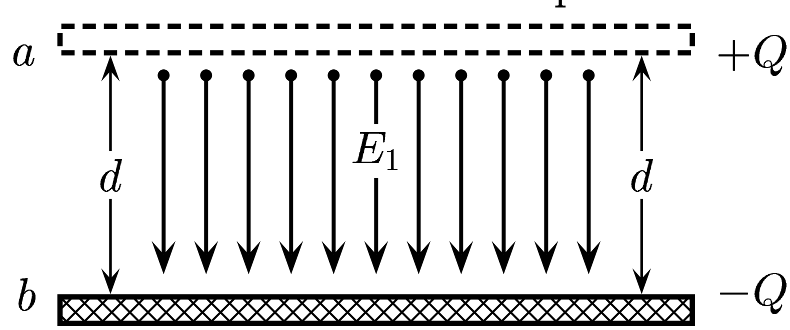


Determine electric force F with which the bottom plate pulls the top plate.

A) $F = Q\, E_{\text{gap}}$

B) $F = \dfrac{1}{2}\, Q\, E_{\text{gap}}$

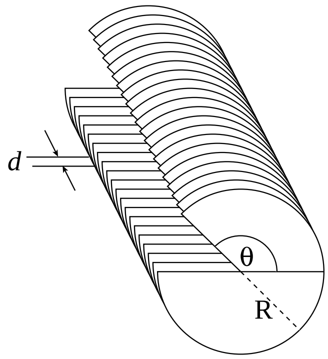
The electric field due to the bottom plate as shown is



$E_1 = \dfrac{Q_{\text{encl}}}{2\,\epsilon_0\, A} = \dfrac{E_{\text{gap}}}{2}$. This leads to $F = Q\, E_1 = \dfrac{Q\, E_{\text{gap}}}{2}$

Answer **B**.

A variable air capacitor used in tuning circuits is made of N semicircular plates each of radius R and positioned d from each other. A second identical set of plates that is free to rotate is enmeshed with the first set.



Determine the capacitance as a function of the angle of rotation θ , where $\theta = 0$ corresponds to the maximum capacitance.

A) $C = \dfrac{\epsilon_0 N R^2 \theta}{d}$

B) $C = \dfrac{\epsilon_0 (2N) R^2 \theta}{d}$

C) $C = \dfrac{\epsilon_0 N R^2 (\pi - \theta)}{d}$

D) $C = \dfrac{\epsilon_0 (2N - 1) R^2 (\pi - \theta)}{d}$

Considering the situation of $\theta = 0$, the two sets of semicircular plates in fact form $2N - 1$ capacitors connected parallel, with each one having capacitance

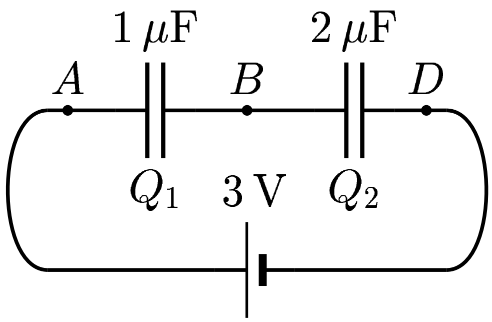
$$C = \frac{\epsilon_0 A}{d/2} = \frac{\epsilon_0 \dfrac{\pi R^2}{2}}{d/2} = \frac{\epsilon_0 \pi R^2}{d}.$$

So the total capacitance would be $(2N - 1)\dfrac{\epsilon_0 \pi R^2}{d}$. *Note:* The common area of the two sets of plates varies linearly when one set is rotating, so the capacitance at angle θ is

$$C = \frac{\epsilon_0 (2N - 1) R^2 (\pi - \theta)}{d}.$$

Answer **D**.

Given $C_1 = 1\,\mu\text{F},\ C_2 = 2\,\mu\text{F},\ \mathcal{E} = 3\,\text{V}$.



Find the ratios $\dfrac{Q_1}{Q_2}$.

A) $\dfrac{Q_1}{Q_2} = 1$

B) $\dfrac{Q_1}{Q_2} = 2$

C) $\dfrac{Q_1}{Q_2} = \dfrac{1}{2}$

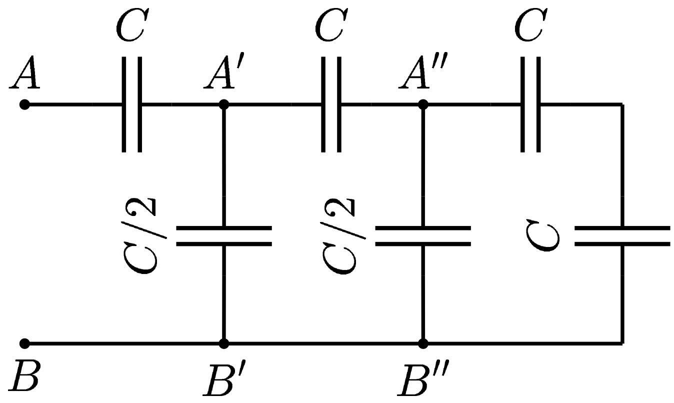
D) $\dfrac{Q_1}{Q_2} = \dfrac{1}{3}$

Note: Each capacitor is a neutral system, so for C_1 and C_2 the corresponding charges must be $(Q_1, -Q_2)$ and $(Q_2, -Q_2)$.

From the figure it follows that $Q_1 = Q_2$.

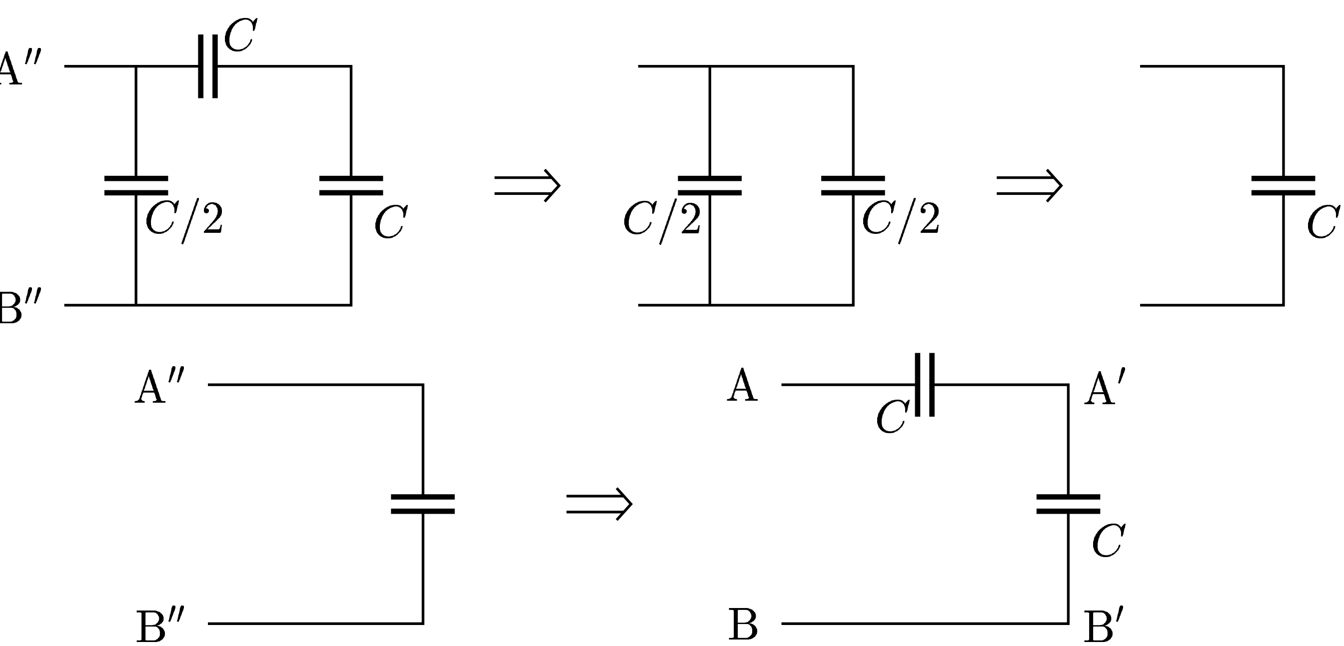
So $\dfrac{Q_1}{Q_2} = 1$.

Answer **A**.



Determine the resultant capacitance C_{AB} for the above network.

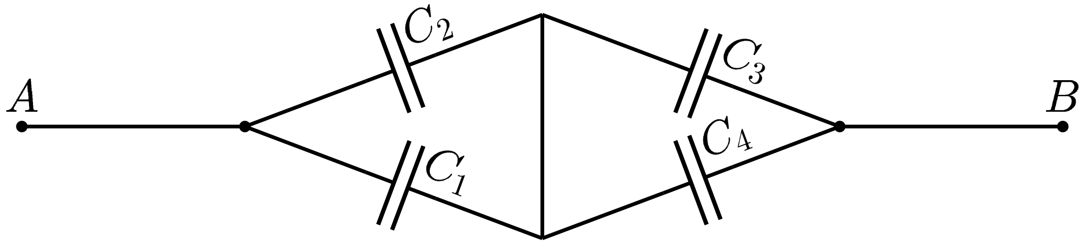
A) $C_{\mathrm{AB}} = \frac{C}{4}$
B) $C_{\mathrm{AB}} = \frac{C}{2}$
C) $C_{\mathrm{AB}} = C$
D) $C_{\mathrm{AB}} = 2\,C$
E) $C_{\mathrm{AB}} = 4\,C$



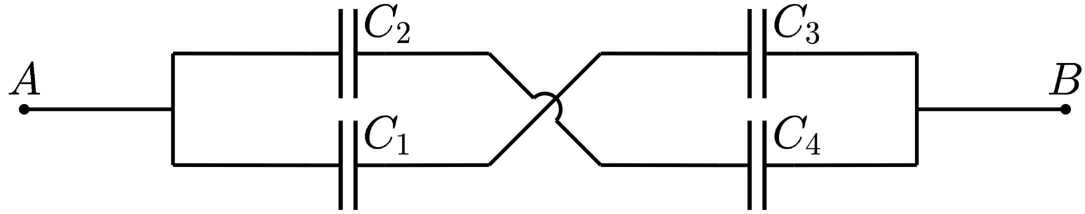
By inspection, $C_{\mathrm{A'B'}} = C_{\mathrm{A''B''}} = C$.
This leads to

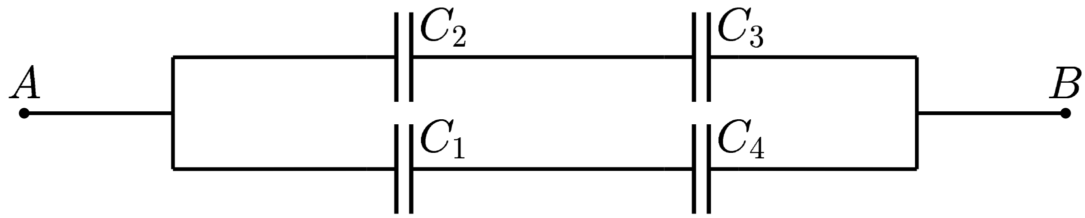
$C_{\mathrm{AB}} = \text{sketch} = \frac{C}{2}$.

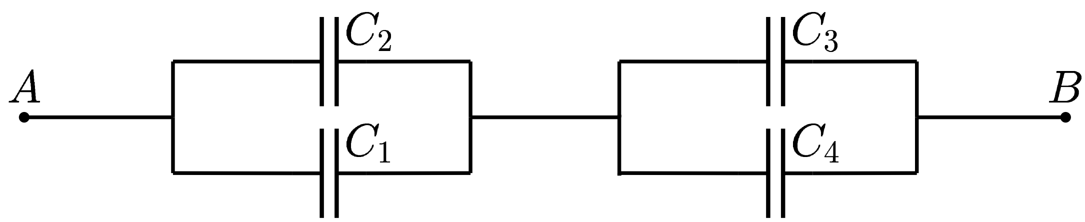
Answer **B**.



Which one of following diagrams represents the same network as one above?

A) 

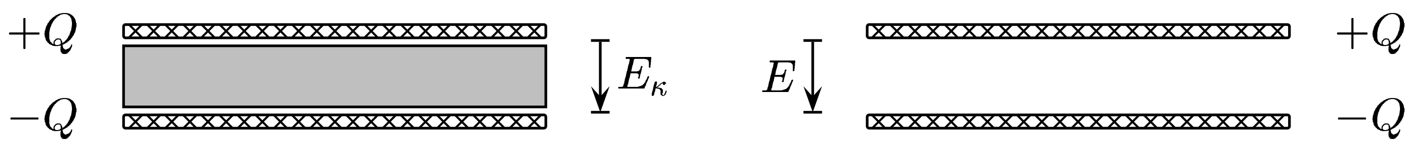
B) 

C) 

Using the deformation rule, one finds that **C** is the match.
Answer **C**.

Two parallel-plate capacitors are shown below. Both are identical except one has a dielectric slab inserted into the gap between the plates. Both capacitors contain the identical charges on their plates.

Hint: $E_\kappa = \dfrac{E}{\kappa}$, $u = \dfrac{U}{A\,d}$, $u_\kappa = \dfrac{U_\kappa}{A\,d}$, $U = \dfrac{Q^2}{2\,C}$, and $U_\kappa = \dfrac{Q^2}{2\,C_\kappa}$.



Find the ratio of the energy densities $\dfrac{u_\kappa}{u}$.

A) $\dfrac{u_\kappa}{u} = 1$

B) $\dfrac{u_\kappa}{u} = \kappa$

C) $\dfrac{u_\kappa}{u} = \dfrac{1}{\kappa}$

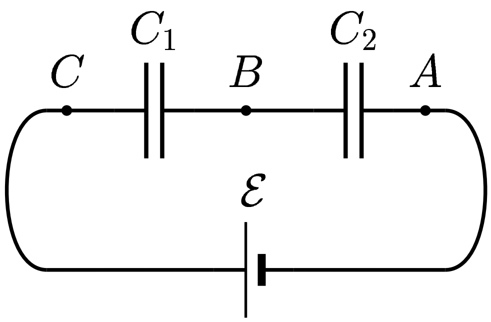
$$u = \frac{U}{A\,d}, \quad \text{and} \quad U = \frac{Q}{2\,C} = \frac{\epsilon_0\,E^2}{2}$$

$$u_\kappa = \frac{U_\kappa}{A\,d}, \quad \text{and} \quad U_\kappa = \frac{Q}{2\,\kappa\,C} = \frac{\epsilon_0\,E^2}{2\,\kappa} = \frac{\epsilon_0\,\kappa^2\,E_\kappa^2}{2\,\kappa} = \kappa\,\frac{\epsilon_0\,E^2}{2}$$

$$\frac{u_\kappa}{u} = \kappa\,.$$

Answer **B**.

Given: Capacitances C_1 and C_2, where $C_2 > C_1$.



Find the comparison between V_{BA} and V_{CB}.

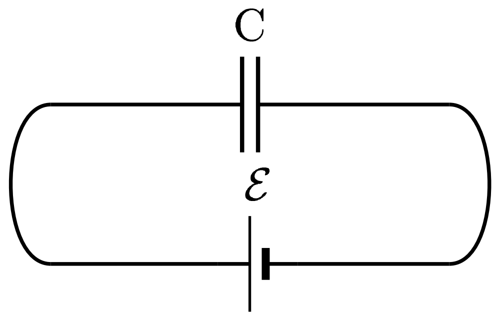
A) $V_{BA} > V_{CB}$
B) $V_{BA} = V_{CB}$
C) $V_{BA} < V_{CB}$
D) Cannot be determined.

$$V \equiv \frac{Q}{C}$$

Therefore, the larger voltage drop will be across the capacitor with the smaller capacitance.

Answer **C**.

A Capacitor with capacitance C is connected to a battery with a voltage V. It has a plate charge $Q$and a total energy U. Fill the gap with material which has dielectric constant κ. The corresponding new quantities are Q' and U'.



Determine the ratio of charges $\dfrac{Q'}{Q}$.

A) $\dfrac{Q'}{Q} = \kappa$

B) $\dfrac{Q'}{Q} = 1$

C) $\dfrac{Q'}{Q} = \dfrac{1}{\kappa}$

$$V' = \frac{Q'}{C'} = V = \frac{Q}{C}, \quad \text{or}$$

$$\frac{Q'}{Q} = \frac{C'}{C} = \kappa .$$

Answer **A**.

26.05-03˙A˙Fixed˙V˙Case 2004-3-24

Consider a typical capacitor, such as a parallel plate capacitor or a spherical capacitor. For each case, capacitance is defined by $C \equiv \dfrac{Q}{V}$. In the presence of a dielectric with a dielectric constant ($\kappa > 1$), while keeping Q fixed, the electric field between the gap will be reduced to $E' = \dfrac{E}{\kappa}$.

Prior to the insertion of a dielectric we have the electric potential V and the capacitance C and after inserting a dielectric we have V' and C', respectively.

Choose the appropriate relationships.

A) $V' = \kappa V$ and $C' = \kappa C$

B) $V' = \kappa V$ and $C' = \dfrac{\kappa}{C}$

C) $V' = \dfrac{V}{\kappa}$ and $C' = \kappa C$

D) $V' = \dfrac{V}{\kappa}$ and $C' = \dfrac{C}{\kappa}$

E) $V' = V$ and $C' = C$

Since $V = E\,d$, we have

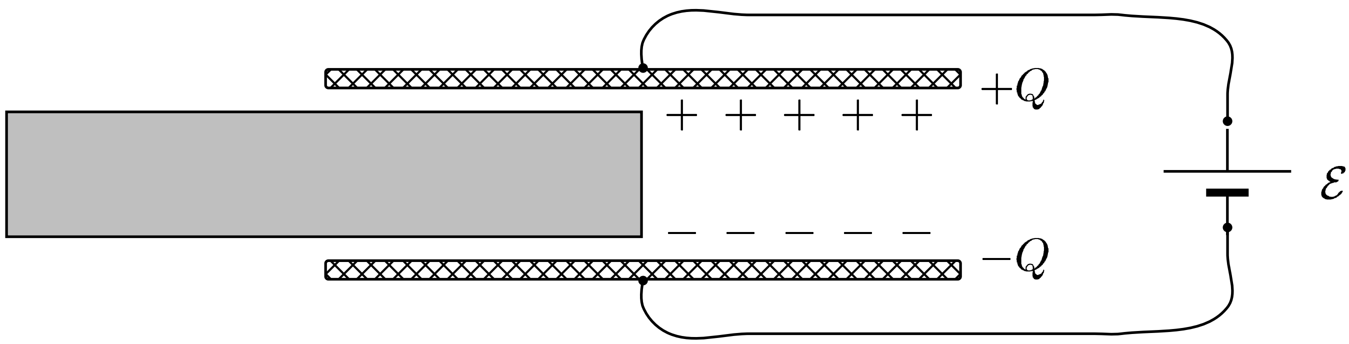
$$V' = E'\,d = \frac{E\,d}{\kappa} = \frac{V}{\kappa},$$

and since $C \equiv \dfrac{Q}{V}$, we have

$$C' = \frac{Q}{V'} = \frac{\kappa\,Q}{V} = \kappa\,C.$$

Answer **C**.

A dielectric slab is half-way into a charged capacitor.



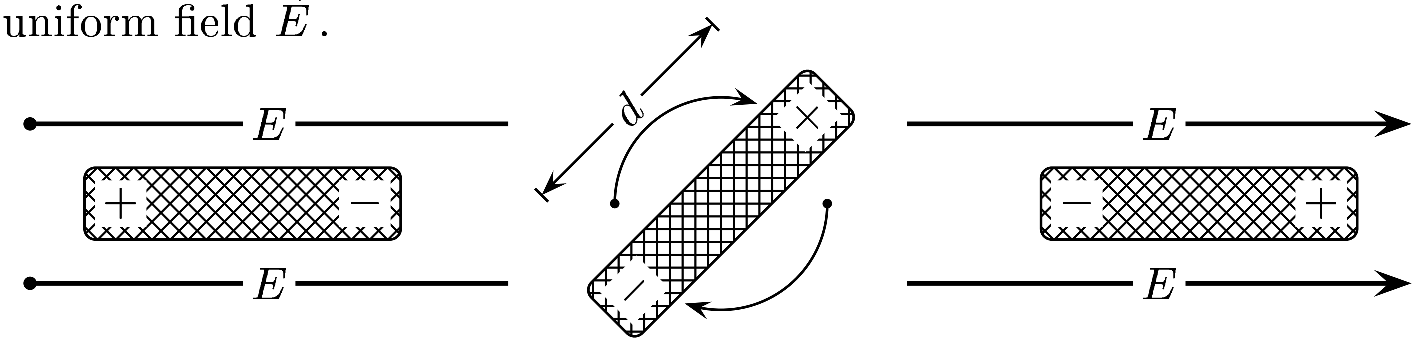
Find the direction of the force on the slab.

A) The direction of the forces is to left.
B) The forces is zero, thus its direction is undetermined.
C) The direction of the force is to right.

Going from no slab to the presence of a slab corresponds to going from a high potential energy to a low potential energy. The natural tendency is to have the slab moving toward filling the gap. In other words, there is an attractive force to the right.

Answer **C**.

A dipole with charge $+q$ and $-q$ separated by a distance d is placed in a uniform field $\vec{E}$.



Determine the net force F_{net} on the dipole, and the potential energy U released in flipping the dipole from the left-hand figure to the right-hand figure.

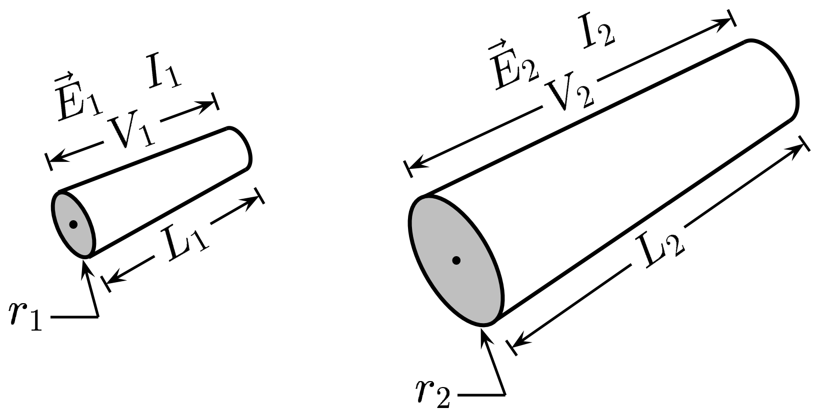
A) $F = 0$ and $|\Delta U| = q\,E\,d$
B) $F = 0$ and $|\Delta U| = 2\,q\,E\,d$
C) $F = 2\,q\,E$ and $|\Delta U| = 2\,q\,E\,d$
D) $F = 2\,q\,E$ and $|\Delta U| = 4\,q\,E\,d$

Electric forces on the two charges asserted by the electric field are equal in magnitude and opposite in direction; *i.e.*, $F = 0$.

For the $+q$ charge the potential energy released is $\Delta U = q\,E\,d$. The $-q$ charge displacement releases the same potential energy; *i.e.*, for both charges $|\Delta U| = 2\,q\,E\,d$.

Answer **B**.

Given : $A = \pi r^2$, $\rho_2 = \rho_1$, $A_2 = 2A_1$, $L_2 = 2L_1$, and $V_2 = V_1$.



Find the ratio $\frac{E_2}{E_1}$ of the electric field in the conductors.

A) $\frac{E_2}{E_1} = 2$.

B) $\frac{E_2}{E_1} = 1$.

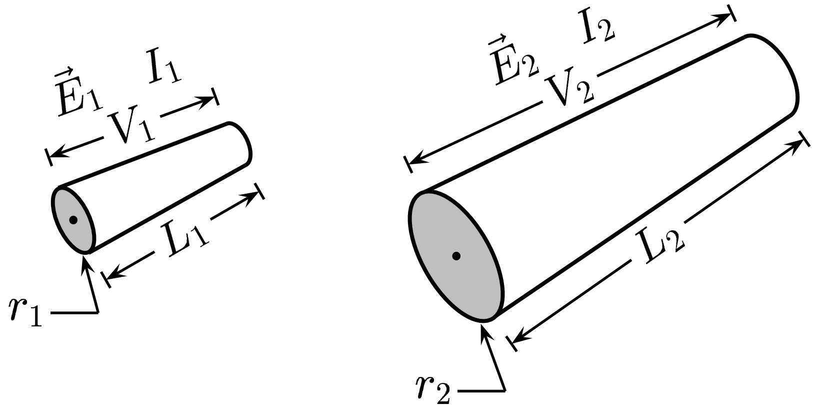
C) $\frac{E_2}{E_1} = \frac{1}{2}$.

Using Ohm's law, we have

$$\frac{E_2}{E_1} = \frac{\left(\frac{V_2}{L_2}\right)}{\left(\frac{V_1}{L_1}\right)} = \frac{L_1}{L_2} = \frac{1}{2}$$

Answer **C**.

Given : $A = \pi r^2$, $\rho_2 = \rho_1$, $A_2 = 2A_1$, $L_2 = 2L_1$, and $V_2 = V_1$.



Find the ratio $\frac{I_2}{I_1}$ of the currents in the conductors.

A) $\frac{I_2}{I_1} = 2$.

B) $\frac{I_2}{I_1} = 1$.

C) $\frac{I_2}{I_1} = \frac{1}{2}$.

Using Ohm's law, we have

$$\frac{I_2}{I_1} = \frac{\left(\frac{V_2}{R_2}\right)}{\left(\frac{V_1}{R_1}\right)} = \frac{R_1}{R_2}$$

$$= \frac{\rho\left(\frac{L_2}{A_2}\right)}{\rho\left(\frac{L_1}{A_1}\right)} = \frac{\left(\frac{L_1}{L_2}\right)}{\left(\frac{A_1}{A_2}\right)} = 1\,.$$

Answer **B**.

Visualize free electrons moving through a crowded medium. They collide with the atoms along the way.

As the temperature increases, what will happen to the average collision time, τ?

What will happen to the resistivity, ρ?

1) τ increases, and ρ increases.
2) τ decreases, and ρ increases.
3) τ increases, and ρ decreases.
4) τ decreases, and ρ decreases.

When the temperature is increased, the atoms in the medium are "vibrating" with faster average speed. Free electrons will collide with atoms more frequently. So the average collision time τ is decreased.

The resistivity $\rho = \dfrac{m}{n\,q^2\,\tau}$; *i.e.*, ρ is inversely proportional to τ. As the collision time decreases, resistivity increases.

Answer 2.

For an ohmic conductor, we have

$$V_{\text{drift}} = a\,\tau,\ a = \frac{q\,E}{m},\ \text{and}\ J = n\,q\,v_{\text{drift}}.$$

So the drift speed or the current density is proportional to E. This relation holds only if τ is strictly constant.

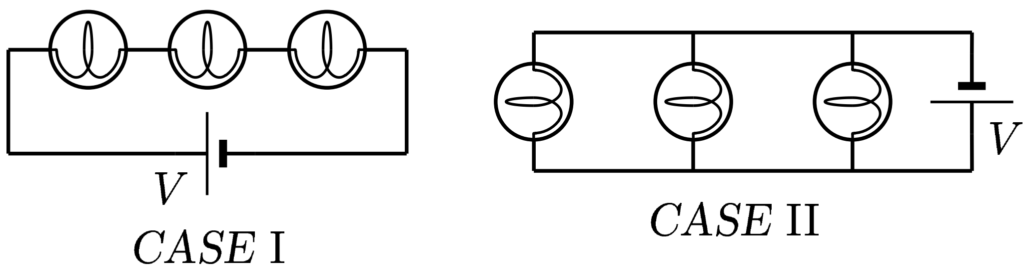
Estimate how much will τ be changed, if v_{drift} is doubled, for a typical case where the average thermal speed is $v_{\text{th}} \approx 10^3\,\text{km/s}$, and average drift speed, $v_{\text{drift}} \approx 10^{-4}\,\text{m/s}$.

1) It increases by more than 0.01%.
2) It stays essentially the same; *i.e.*, the change is less than 0.01%.
3) In decreases by more than 0.01%.

Based on thermo energy considerations the reader should convince himself/herself that the percentage of the maximum change in in v is less than $\dfrac{2\,v_{\text{drift}}}{v_{\text{th}}} = \dfrac{2\times10^{-4}}{10^6} = 2\times10^{-8}\%$. This is less than 0.01%.

Answer 2.

Three identical bulbs are connected in two ways as shown.



Determine $\frac{P_{\mathrm{II}}}{P_{\mathrm{I}}}$, where P_{I} is the power per bulb in *CASE* I, and P_{II} is in *CASE* II.

A) $\frac{P_{\mathrm{II}}}{P_{\mathrm{I}}} = 9$

B) $\frac{P_{\mathrm{II}}}{P_{\mathrm{I}}} = 3$

C) $\frac{P_{\mathrm{II}}}{P_{\mathrm{I}}} = \frac{1}{3}$

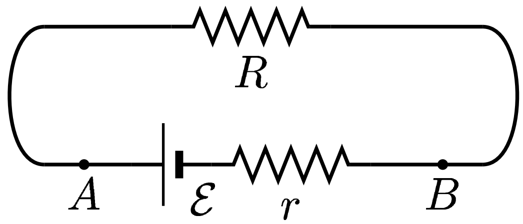
D) $\frac{P_{\mathrm{II}}}{P_{\mathrm{I}}} = \frac{1}{9}$

$$\frac{P_{\mathrm{II}}}{P_{\mathrm{I}}} = \frac{\frac{V^2}{R}}{\frac{(V/3)^2}{R}} = 9$$

.

Answer **A**.

Given battery has *emf* $\mathcal{E} = 10$ V and the internal resistance $r = 1\,\Omega$, as shown in the figure below. An external resistance $R = 0.01\ \Omega$ is connected to the battery.



Compare V_{AB} with $\mathcal{E}$.

A) $V_{AB} \ll \mathcal{E}$.
B) $V_{AB} \approx \mathcal{E}$.
C) $V_{AB} \gg \mathcal{E}$.

$$R_{total} = R + r$$

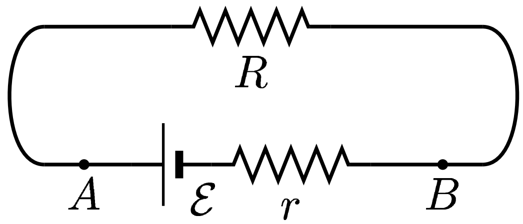
$$I = \frac{\mathcal{E}}{R + r}$$

$$V_{AB} = \frac{R}{R + r}\,\mathcal{E} = \frac{0.01}{0.01 + 1}\,10\text{ V} = .099\text{ V}$$

This simple calculation shows $V_{AB} = .099\,\text{V} \ll \mathcal{E}$. In other words, when $r \gg R$, most of the potential drop is across the internal resistance r.

Answer **A**.

Given battery has *emf* $\mathcal{E} = 10$ V and the internal resistance $r = 1\,\Omega$, as shown in the figure below. An external resistance $R = 100.0\,\Omega$ is connected to the battery.



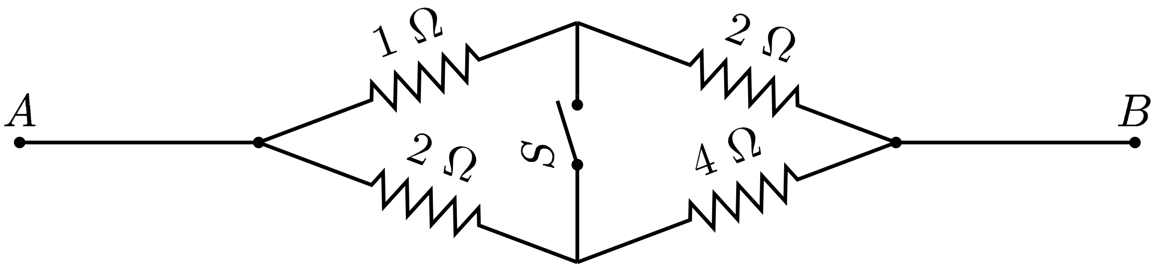
Compare V_{AB} with $\mathcal{E}$.

A) $V_{AB} \ll \mathcal{E}$.
B) $V_{AB} \approx \mathcal{E}$.
C) $V_{AB} \gg \mathcal{E}$.

$$R_{total} = R + r$$
$$I = \frac{\mathcal{E}}{R + r}$$
$$V_{AB} = \frac{R}{R + r}\,\mathcal{E} = \frac{100}{100 + 1}\,10\text{ V} = 9.9\text{ V}$$

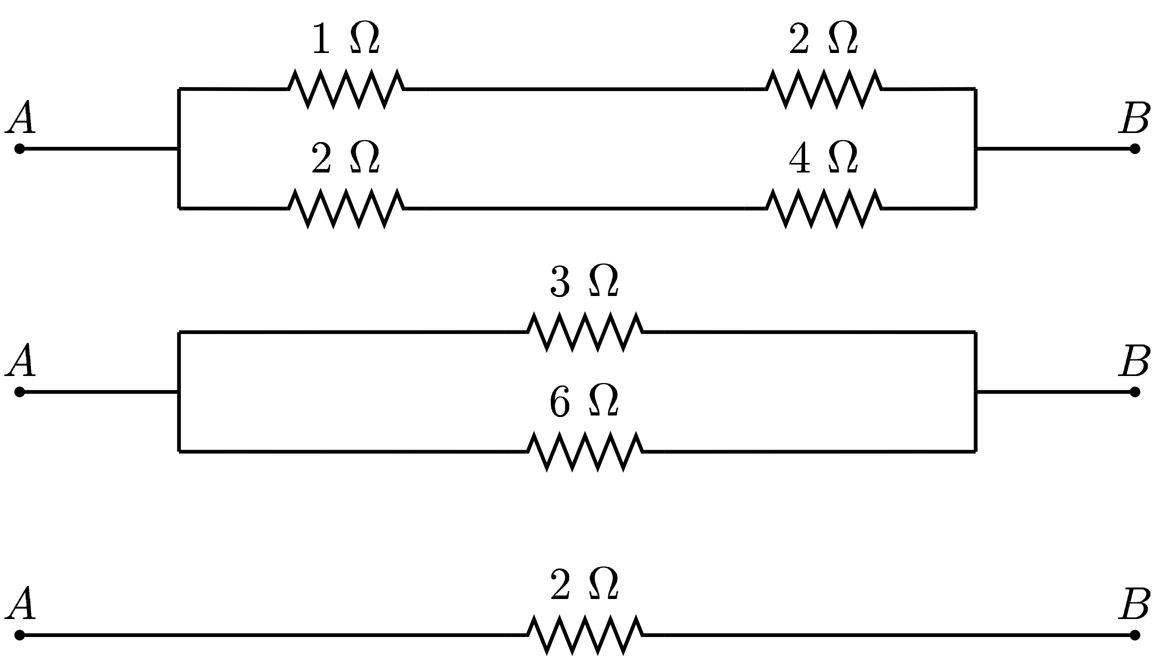
This simple calculation shows $V_{AB} = 9.9\,\text{V} \approx \mathcal{E}$. In other words, when $r \ll R$, most of the potential drop is across the external resistance R.

Answer **B**.



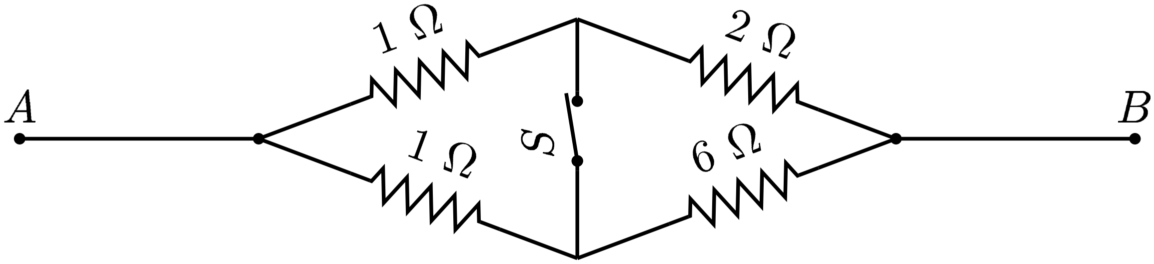
When the switch is open, what is the resistance R_{AB}?

A) $R_{AB} = \frac{1}{4}\ \Omega$
B) $R_{AB} = \frac{1}{2}\ \Omega$
C) $R_{AB} = 1.0\ \Omega$
D) $R_{AB} = 2.0\ \Omega$
E) $R_{AB} = 4.0\ \Omega$



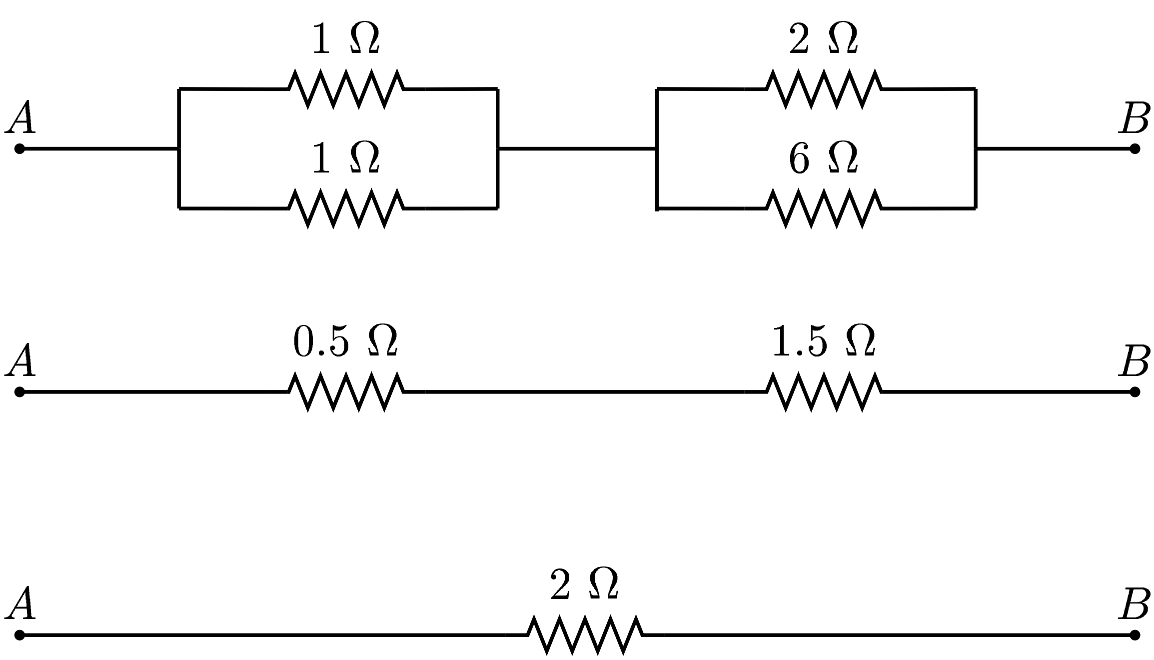
Answer **D**.

28.02-02a˙Equivalent˙Diagram 2004-3-24

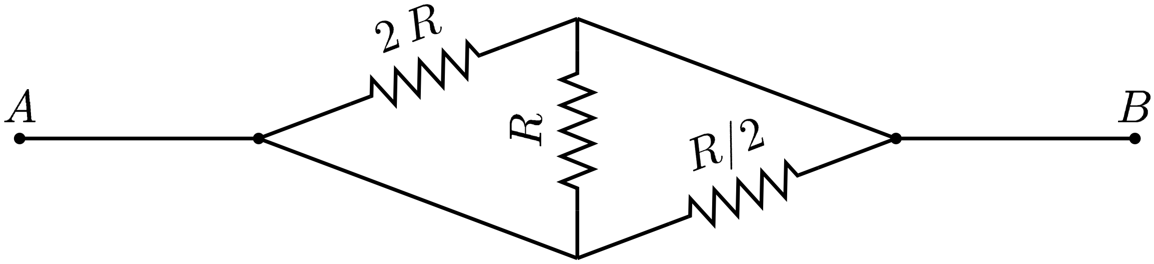


When the switch is closed, what is the resistance R_{AB}?

A) $R_{AB} = \frac{1}{6}\ \Omega$
B) $R_{AB} = \frac{1}{2}\ \Omega$
C) $R_{AB} = 1.0\ \Omega$
D) $R_{AB} = 2.0\ \Omega$
E) $R_{AB} = 4.0\ \Omega$



Answer **D**.



What is the resistance R_{AB}?

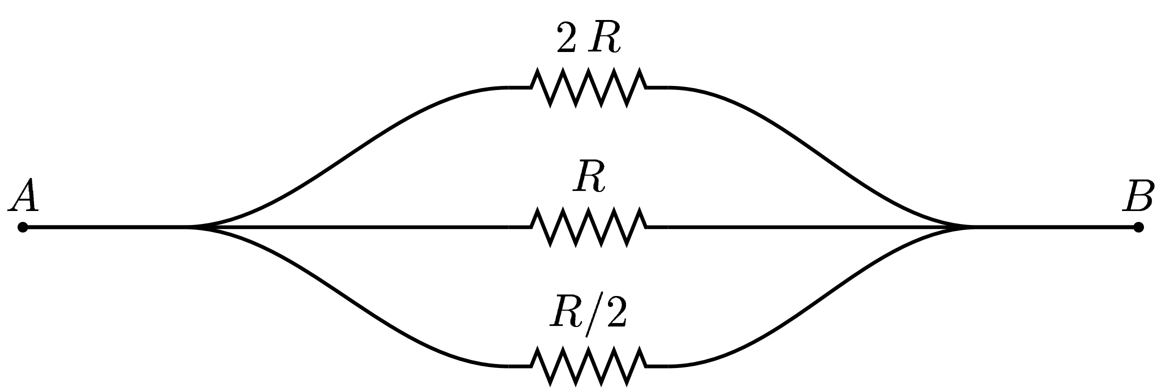
A) $R_{AB} = \frac{2}{3}\, R$

B) $R_{AB} = \frac{3}{5}\, R$

C) $R_{AB} = \frac{2}{5}\, R$

D) $R_{AB} = \frac{3}{7}\, R$

E) $R_{AB} = \frac{2}{7}\, R$

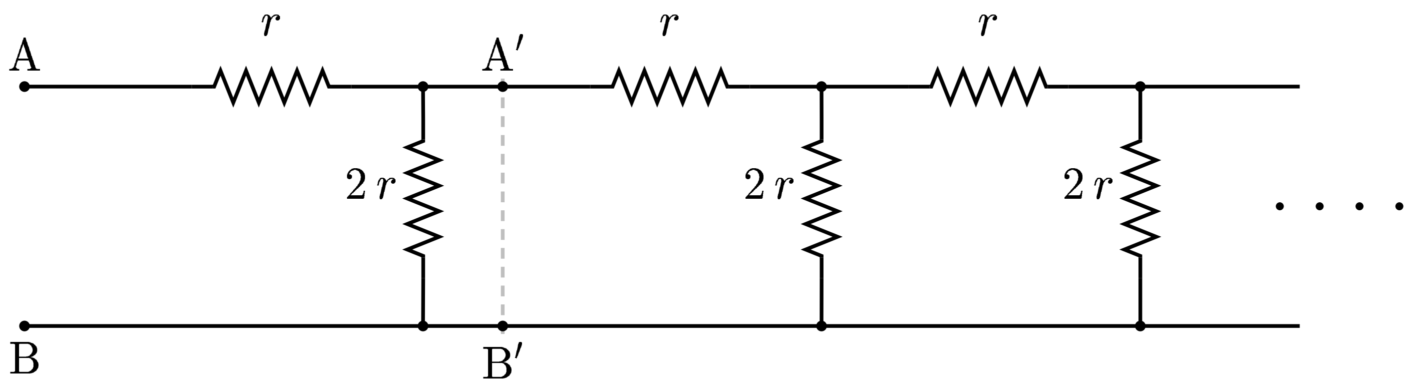


$$R_{AB} = \frac{1}{\frac{1}{2\,R} + \frac{1}{R} + \frac{2}{R}} = \frac{1}{\frac{1}{2\,R} + \frac{2}{2\,R} + \frac{4}{2\,R}} = \frac{2}{7}\, R\,.$$

Answer **E**.

Hint: The resistance to the right of A′B′ is the same as the resistance to the right of AB; that is, $R_{\mathrm{AB}} = R_{\overrightarrow{\mathrm{A'B'}}}$.

Given an infinite chain with a repetitive pattern as shown.



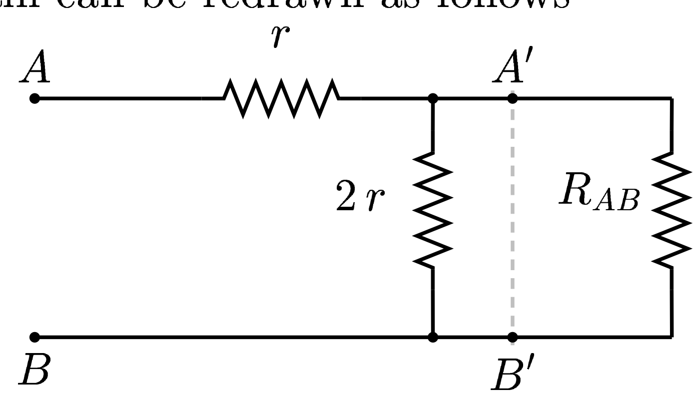
Select an equation with R_{AB} and r which can be used to solve for R_{AB} in terms of r.

1) $R_{AB} = r + \dfrac{2\,r\,R_{AB}}{2\,r + R_{AB}}$, therefore $R_{AB} = 2\,r$

2) $R_{AB} = r + \dfrac{2\,r\,R_{AB}}{r + R_{AB}}$, therefore $R_{AB} = \dfrac{5}{2}\,r$

3) $R_{AB} = r + 2\,r - R_{AB}$, therefore $R_{AB} = \dfrac{3}{2}\,r$

So the infinite chain can be redrawn as follows



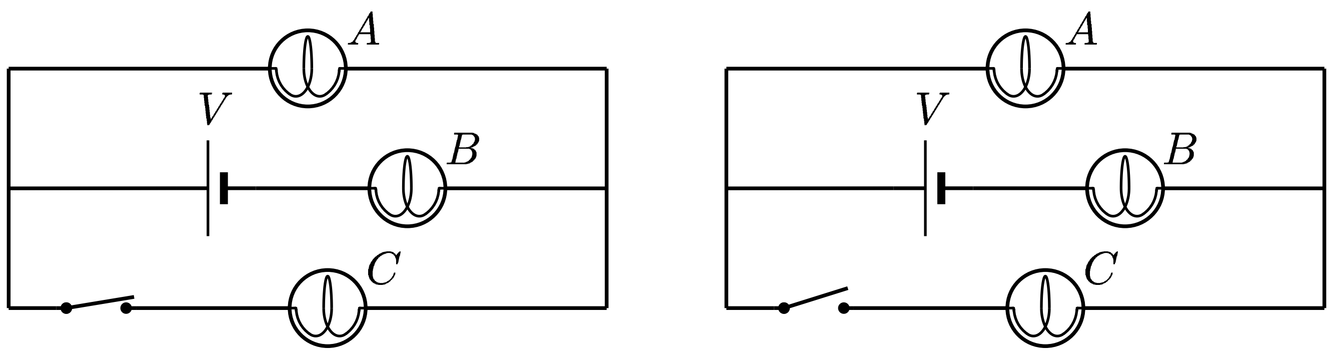
$$R_{AB} = r + \frac{1}{\dfrac{1}{2\,r} + \dfrac{1}{R_{AB}}} = r + \frac{2\,r\,R_{AB}}{R_{AB} + 2\,r}$$

$$R_{AB}^2 - r\,R_{AB} - 2\,r = 0$$

$$R_{AB} = \frac{r \pm \sqrt{r^2 + 4\,(2\,r)}}{2} = \frac{r + 3\,r}{2} = 2\,r\,.$$

Answer **A**.

Three identical bulbs are connected in two ways as shown. Denote the current I when the switch is closed and I' when the switch is open.



The ratio of currents through the bulb B is given by

A) $\frac{I'_B}{I_B} = \frac{4}{3}$

B) $\frac{I'_B}{I_B} = 1$

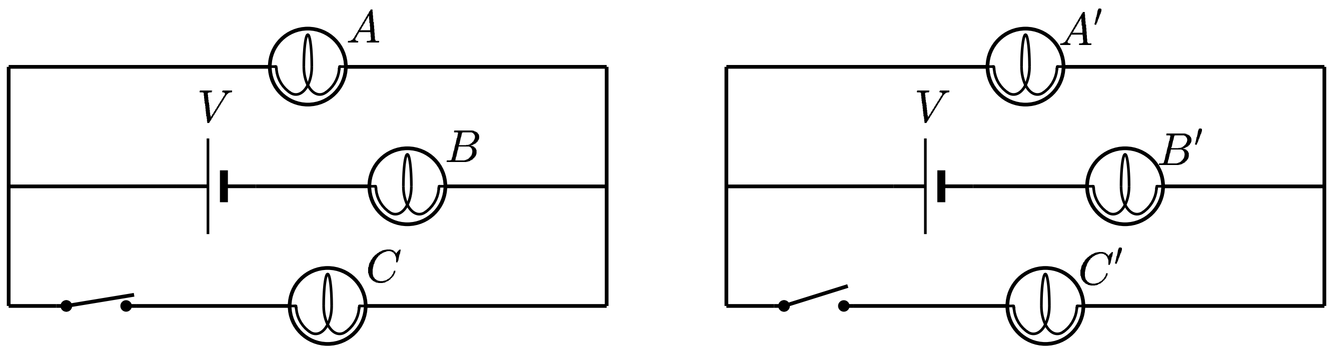
C) $\frac{I'_B}{I_B} = \frac{3}{4}$

D) $\frac{I'_B}{I_B} = \frac{2}{3}$

$$\frac{I'_B}{I_B} = \frac{\frac{V}{2\,R}}{\frac{V}{\frac{3\,R}{2}}} = \frac{3}{4}$$

Answer **C**.

Three identical bulbs are connected in two ways as shown. Denote the brightness without a prime when the switch is closed and with a prime ′ when the switch is open.



Compare the respective brightnesses of bulbs A and B when the switch is closed to when the switch is open.

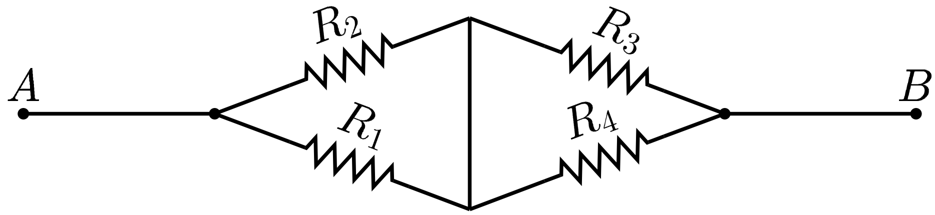
A) $B' > B$ and $A' > A$
B) $B' < B$ and $A' > A$
C) $B' > B$ and $A' < A$
C) $B' < B$ and $A' < A$

$$\frac{I'_B}{I_B} = \frac{\dfrac{V}{2\,R}}{\dfrac{V}{\dfrac{3\,R}{2}}} = \frac{3}{4}$$

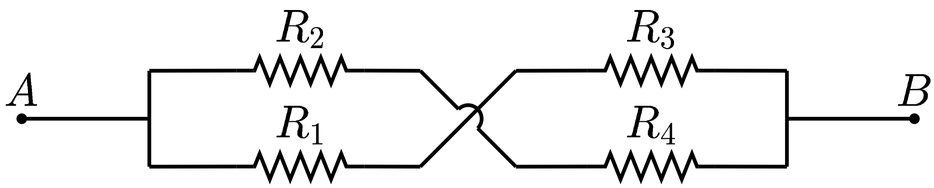
$$\frac{I'_A}{I_A} = \frac{\dfrac{V}{2\,R}}{\dfrac{1}{2}\,\dfrac{V}{\dfrac{3\,R}{2}}} = \frac{3}{2}$$

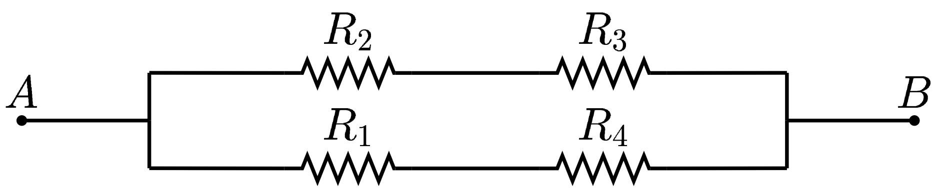
Since the brightness is directly proportional to the power $P = I^2\,R$, $B' < B$ and $A' > A$.

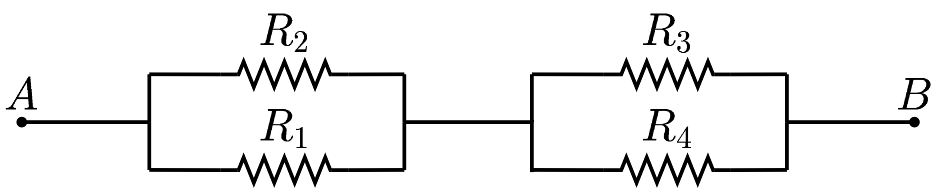
Answer **B**.



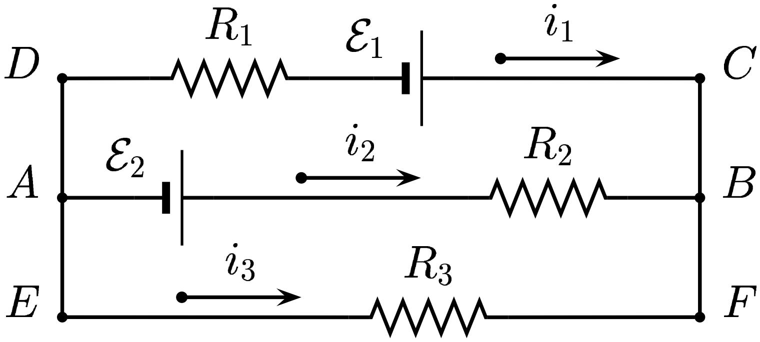
Which one of following diagrams represents the same network as the one above?

A) 

B) 

C) 

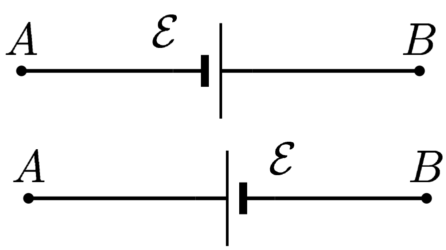
Using the deformation rule, one finds that **C** is the match.
Answer **C**.



Find the loop equation for the loop $ABFEA$.

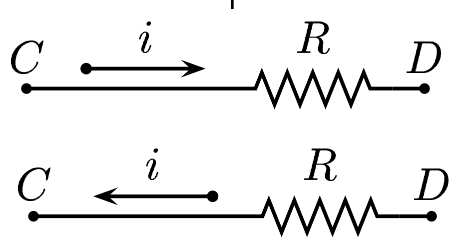
A) $\mathcal{E}_2 - i_2\,R_2 - i_3\,R_3 = 0$
B) $\mathcal{E}_2 + i_2\,R_2 - i_3\,R_3 = 0$
C) $\mathcal{E}_2 - i_2\,R_2 + i_3\,R_3 = 0$
D) $\mathcal{E}_2 + i_2\,R_2 - i_3\,R_3 = 0$

For loop $ABFEA$, we have $\mathcal{E}_2 - i_2\,R_2 + i_3\,R_3 = 0$

Convention 1: 

$\Delta V = V_B - V_A = +\,\mathcal{E}$

$\Delta V = V_B - V_A = -\,\mathcal{E}$

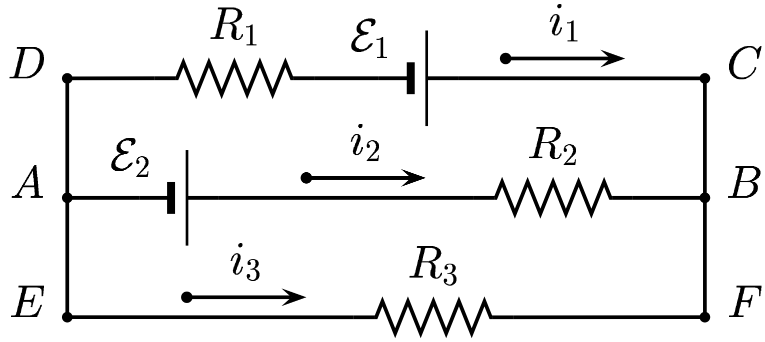
Convention 2: 

$\Delta V = V_D - V_C = -\,i\,R$

$\Delta V = V_D - V_C = +\,i\,R$

Convention 3: Currents into a junction are positive and currents out of a junction are negative.

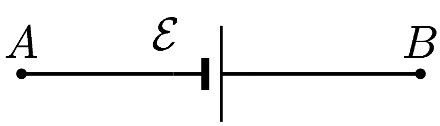
Answer **C**.

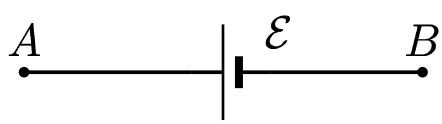


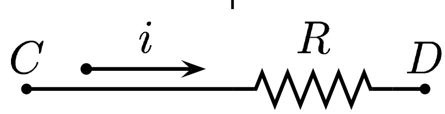
Find the loop equation for the loop $ABCDA$.

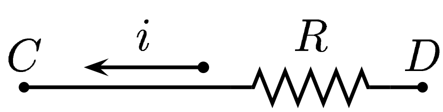
A) $\mathcal{E}_2 + i_2 R_2 - \mathcal{E}_1 + i_1 R_1 = 0$
B) $\mathcal{E}_2 - i_2 R_2 + \mathcal{E}_1 - i_1 R_1 = 0$
C) $\mathcal{E}_2 + i_2 R_2 - \mathcal{E}_1 - i_1 R_1 = 0$
D) $\mathcal{E}_2 + i_2 R_2 + \mathcal{E}_1 - i_1 R_1 = 0$
E) $\mathcal{E}_2 - i_2 R_2 - \mathcal{E}_1 + i_1 R_1 = 0$

For loop $ABCDA$, we have $\mathcal{E}_2 - i_2 R_2 - \mathcal{E}_1 + i_1 R_1 = 0$

Convention 1:  $\Delta V = V_B - V_A = +\mathcal{E}$

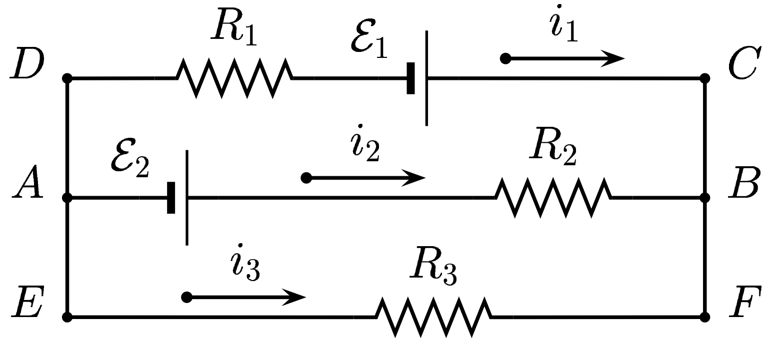
 $\Delta V = V_B - V_A = -\mathcal{E}$

Convention 2:  $\Delta V = V_D - V_C = -i R$

 $\Delta V = V_D - V_C = +i R$

Convention 3: Currents into a junction are positive and currents out of a junction are negative.

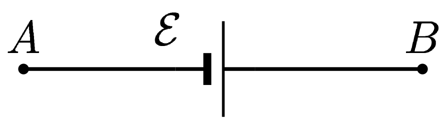
Answer **E**.

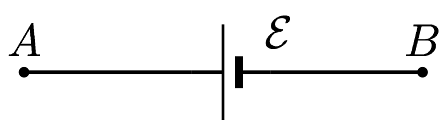


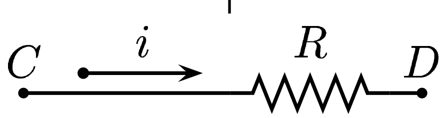
Find the node equation for the junction A.

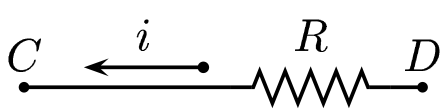
A) $i_1 + i_2 + i_3 = 0$
B) $i_1 - i_2 + i_3 = 0$
C) $i_1 + i_2 - i_3 = 0$
D) $i_1 - i_2 - i_3 = 0$

For the junction A, the sum of the currents exiting minus the sum of the currents entering the junction is zero, $i_1 + i_2 + i_3 = 0$.

Convention 1:  $\Delta V = V_B - V_A = +\mathcal{E}$

 $\Delta V = V_B - V_A = -\mathcal{E}$

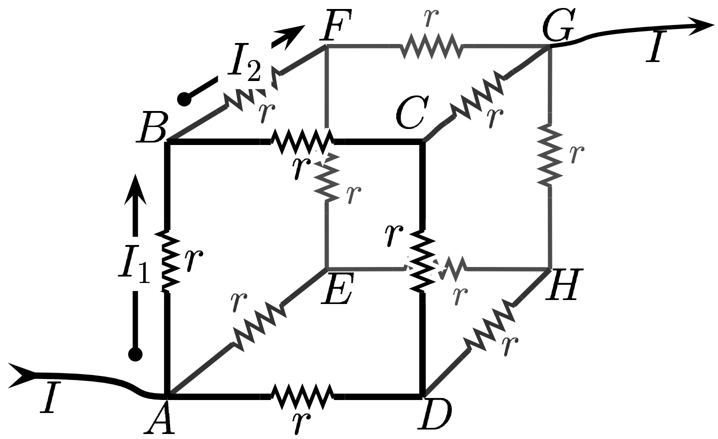
Convention 2:  $\Delta V = V_D - V_C = -i\,R$

 $\Delta V = V_D - V_C = +i\,R$

Convention 3: Currents into a junction are positive and currents out of a junction are negative.

Answer **A**.

Given: A cubic network has identical resistors, each with a resistance r. A current I enters the network at A and leaves at G.



Find current I_1 and I_2 in terms of the total current I through the network.

A) $I_1 = \dfrac{I}{2}$ and $I_2 = \dfrac{I}{4}$.

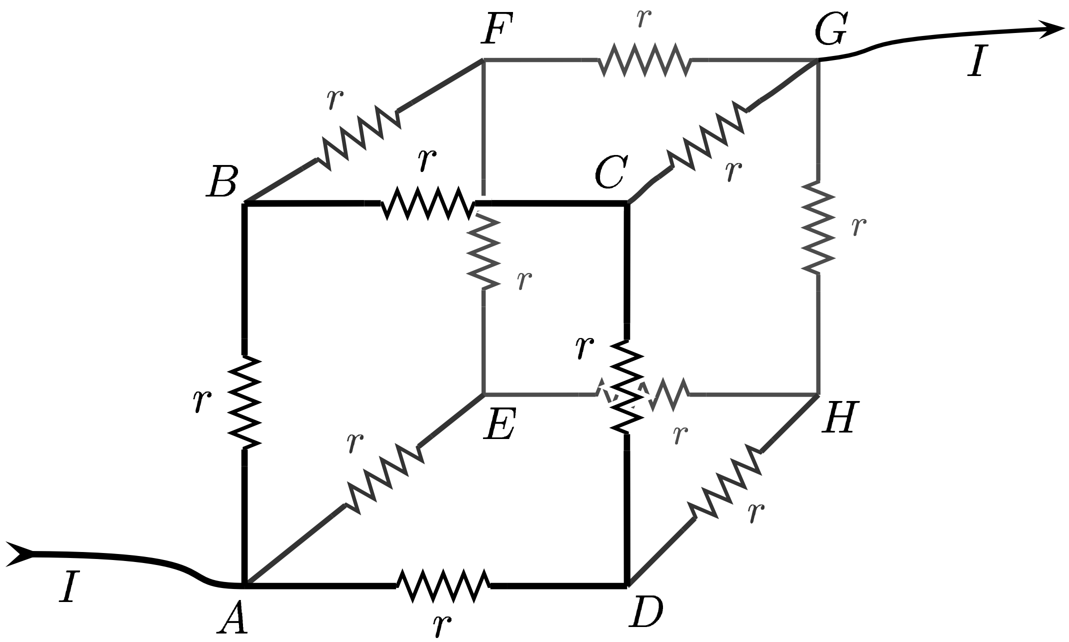
B) $I_1 = \dfrac{I}{3}$ and $I_2 = \dfrac{I}{3}$.

C) $I_1 = \dfrac{I}{3}$ and $I_2 = \dfrac{I}{6}$.

By symmetry, at A, I is equally divided into 3 equal branches. So $I_1 = \dfrac{I}{3}$. By symmetry, at B, I_1 is equally divided into 2 equal branches. So $I_2 = \dfrac{I_1}{2} = \dfrac{I}{6}$.

Answer **C**.

Given: A cubic network has identical resistors, each with a resistance r. A current I enters the network at A and leaves at G.

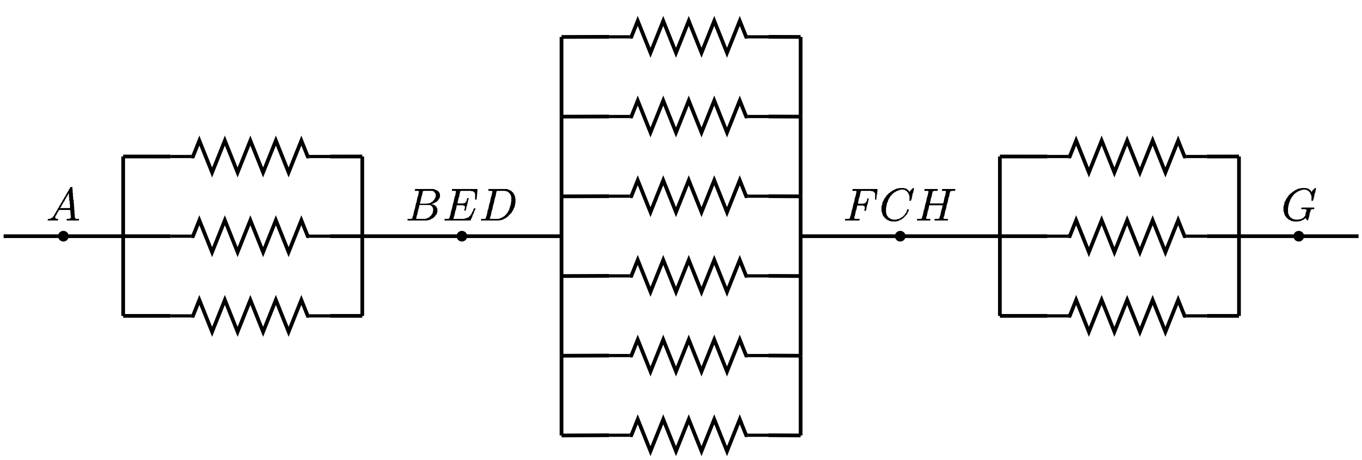


Find the network resistance r_{total} in terms an individual resistor r.

A) $r_{total} = \dfrac{2\,r}{3}$ B) $r_{total} = r$ C) $r_{total} = 2\,r$

D) $r_{total} = \dfrac{4\,r}{3}$ E) $r_{total} = \dfrac{5\,r}{6}$

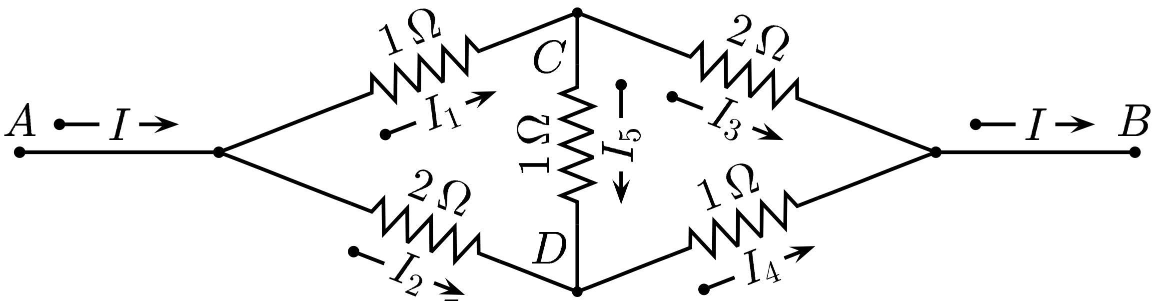
By symmetry, at A, I is equally divided into 3 equal branches and the potential at the junctions B, E, and D are the same, the these points can be joined together without changing the network resistance r_{total}. The same is true at the junctions F, C, and H. The redrawn network is shown below.



Answer **E**. $r_{total} = \dfrac{1\,r}{3} + \dfrac{1\,r}{6} + \dfrac{1\,r}{3} = \dfrac{5\,r}{6}$.

28.03-05˙A˙Cubic˙Network 2004-10-22

The current enters at A and leaves at B.



Determine the ratio $\dfrac{I_2}{I_1}$.

A) $\dfrac{I_2}{I_1} = \dfrac{1}{2}$

B) $\dfrac{I_2}{I_1} = \dfrac{1}{3}$

C) $\dfrac{I_2}{I_1} = \dfrac{2}{3}$

D) $\dfrac{I_2}{I_1} = 1$

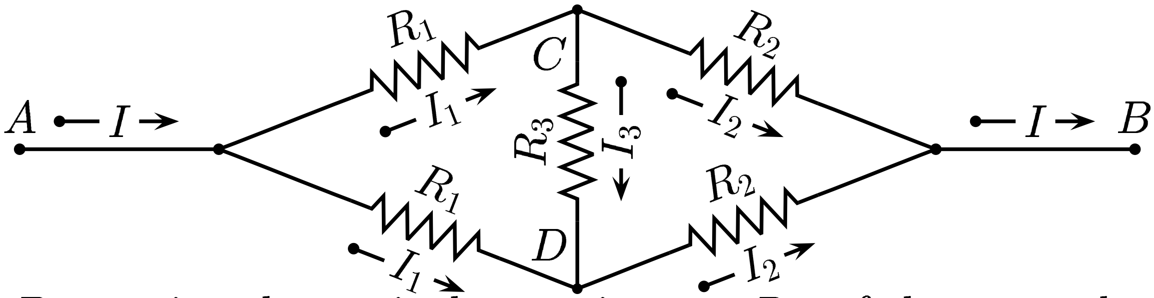
Hint: From symmetry, $I_1 = I_4$. Then for the junction equation $I_5 = I_4 - I_2 = I_1 - I_3$. Write down the loop equation for $ACDA$.

The left-hand loop equation is

$$\begin{aligned} -I_1\,(1\,\Omega) - I_5\,(1\,\Omega) + I_2\,(2\,\Omega) &= 0 \\ -I_1\,(1\,\Omega) - (I_4 - I_2)\,(1\,\Omega) + 2\,I_2\,(1\,\Omega) &= 0 \\ -I_1\,(1\,\Omega) - I_4\,(1\,\Omega) + I_2\,(1\,\Omega) + 2\,I_2\,(1\,\Omega) &= 0 \\ -I_1\,(1\,\Omega) - I_1\,(1\,\Omega) + 3\,I_2\,(1\,\Omega) &= 0 \\ -2\,I_1\,(1\,\Omega) + 3\,I_2\,(1\,\Omega) &= 0 \\ +3\,I_2\,(1\,\Omega) &= 2\,I_1\,(1\,\Omega) \\ \frac{I_2}{I_1} &= \frac{2\,(1\,\Omega)}{3\,(1\,\Omega)} \\ &= \frac{2}{3}\,. \end{aligned}$$

Answer **C**.

The current enters at A and leaves at B.



Determine the equivalent resistance R_{eq} of the network.

A) $R_{eq} = R_1 + R_2$

B) $R_{eq} = \frac{1}{2}\,(R_1 + R_2)$

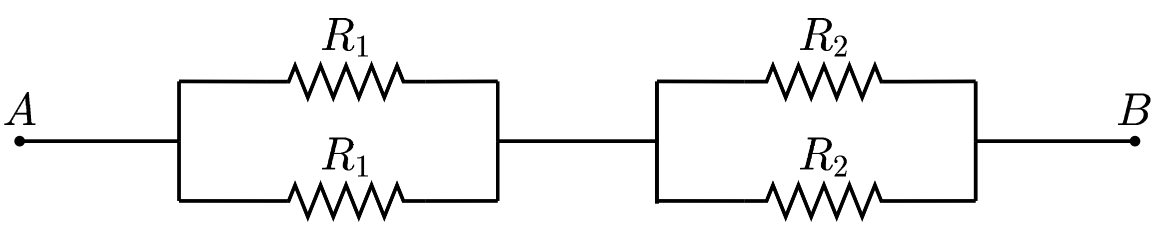
C) $R_{eq} = \dfrac{R_1\,R_2 + R_1\,R_3 + R_2\,R_3}{R_1 + R_2}$

D) $R_{eq} = \dfrac{1}{2}\,\dfrac{R_1\,R_2 + R_1\,R_3 + R_2\,R_3}{R_1 + R_2}$

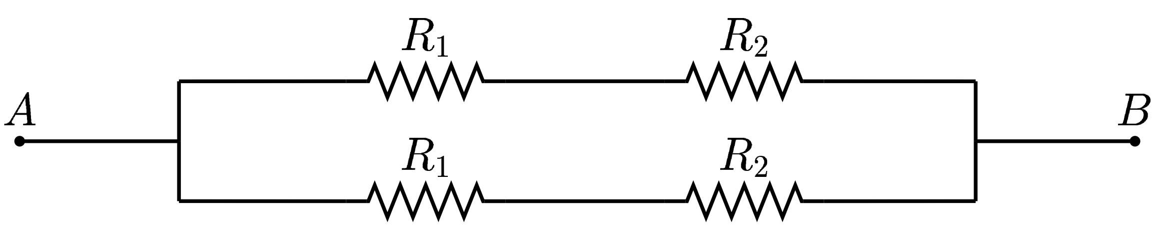
E) $R_{eq} = \dfrac{1}{2}\,\dfrac{R_1\,R_2 + R_1\,R_3 + R_2\,R_3}{R_3}$

Hint: The network is symmetric.

The left-hand and right-hand loop equations are
$-I_1\,R_1 - I_3\,R_3 + I_1\,R_1 = 0$ and also $-I_2\,R_2 + I_2\,R_2 + I_3\,R_3 = 0$ thus $I_5 = 0$.



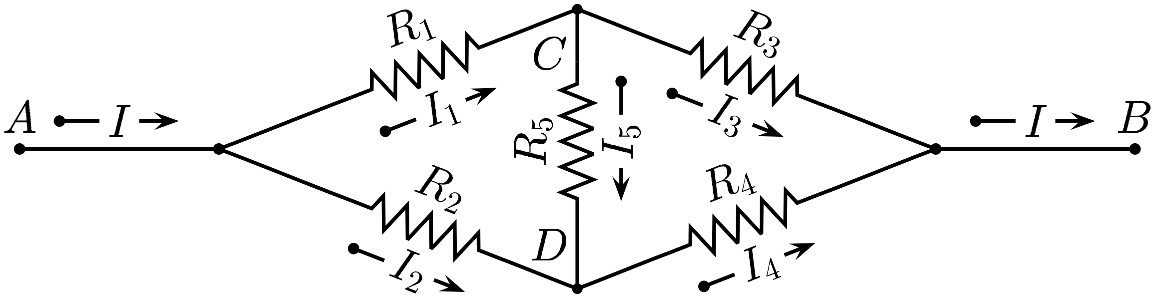
$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_1}} + \frac{1}{\frac{1}{R_2} + \frac{1}{R_2}} = \frac{1}{\frac{2}{R_1}} + \frac{1}{\frac{2}{R_2}} = \frac{1}{2}\,R_1 + \frac{1}{2}\,R_2 = \frac{1}{2}\,(R_1 + R_2)$$



$$R_{eq} = \frac{1}{\frac{1}{R_1 + R_2} + \frac{1}{R_1 + R_2}} = \frac{1}{\frac{2}{R_1 + R_2}} = \frac{1}{2}\,(R_1 + R_2)$$

Answer **C**.

The current enters at A and leaves at B.



The loop equations $ACDA$ is $-I_1\,R_1 - I_5\,R_5 + I_2\,R_2 = 0\,.$

Determine the equation for the loop $CDBC$ in terms of I_1, I_2, and I_5.

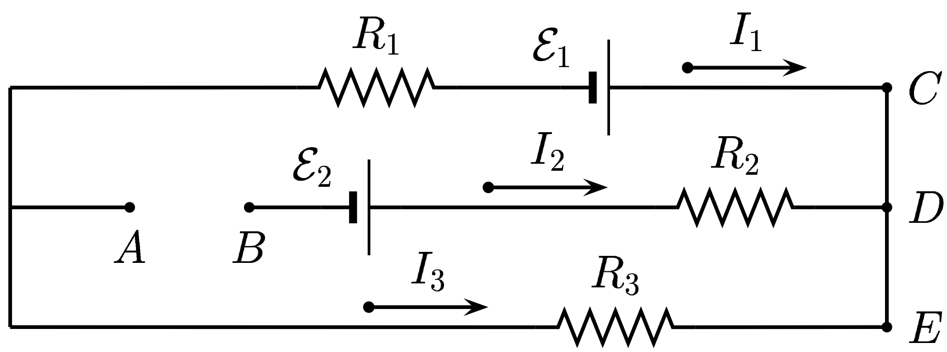
A) $(I_1 - I_2)\,R_3 - (I_2 + I_5)\,R_4 + I_5\,R_5 = 0$
B) $(I_1 - I_3)\,R_3 - (I_2 - I_5)\,R_4 + I_5\,R_5 = 0$
C) $-(I_1 - I_5)\,R_3 + (I_2 + I_5)\,R_4 + I_5\,R_5 = 0$
D) $(I_5 - I_1)\,R_3 + (I_2 - I_5)\,R_4 + I_5\,R_5 = 0$
E) $(I_1 + I_5)\,R_3 - (I_5 - I_2)\,R_4 - I_5\,R_5 = 0$

Hint: Use the node equations $I_3 = I_1 - I_5$ and $I_4 = I_2 + I_5$.

The right-hand loop equation is

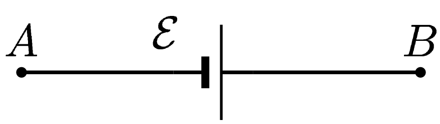
$$-I_3\,R_3 + I_4\,R_4 + I_5\,R_5 = 0$$
$$-(I_1 - I_5)\,R_3 + (I_2 + I_5)\,R_4 + I_5\,R_5 = 0\,.$$

Answer **C**.

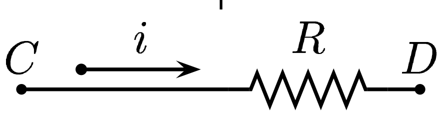


Find the potential difference $V_A - V_B$.

A) $V_A - V_B = \mathcal{E}_2 + \mathcal{E}_1 + R_1\, I_1$
B) $V_A - V_B = \mathcal{E}_2 - I_2\, R_2 + \mathcal{E}_1 - R_1\, I_1$
C) $V_A - V_B = \mathcal{E}_2 + I_2\, R_2 - I_3\, R_3$
D) $V_A - V_B = \mathcal{E}_2 + I_3\, R_3$
E) $V_A - V_B = \mathcal{E}_2 + I_2\, R_2 + R_1\, I_1$

Convention 1: 

$\Delta V = V_B - V_A = +\,\mathcal{E}$

Convention 2: 

$\Delta V = V_D - V_C = -\,i\,R$

For loop $BDEA$

$$V_A - V_B = \mathcal{E}_2 - I_2\, R_2 + I_3\, R_3$$
$$= \mathcal{E}_2 + I_3\, R_3\,,$$

since $I_2 = 0$.

Answer **D**.